<2024 WPTCE>

General Dynamic Gyrator Models for Transient Analysis of IPT

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TESLA Mobile Power Electronics

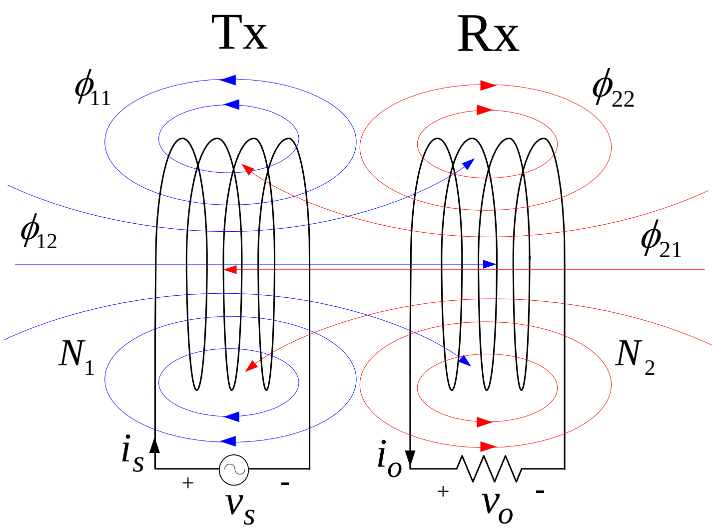
광주과학기술원

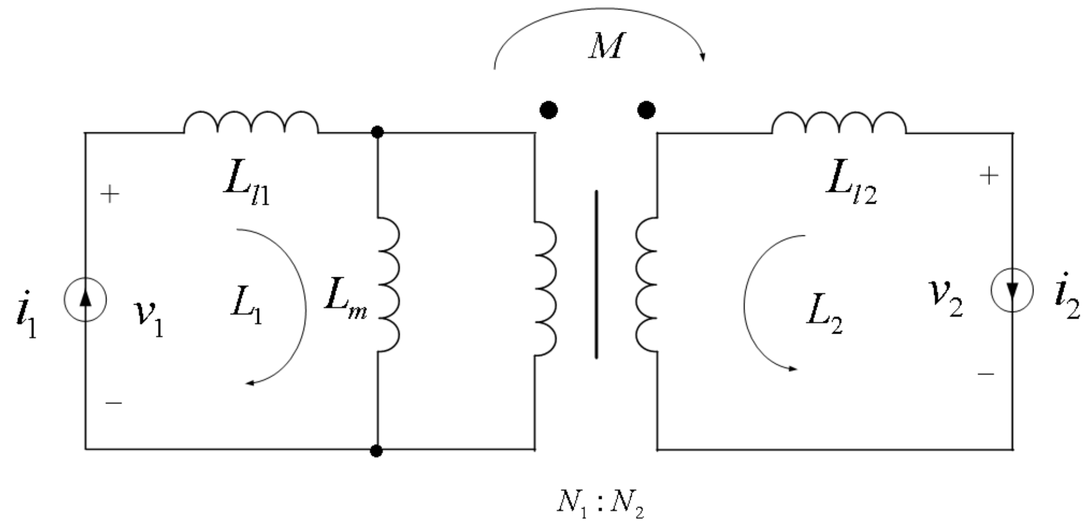
Contents

I. Introduction

Principle of Inductive Power Transfer (IPT)

- IPT is a sort of "coupled inductors"



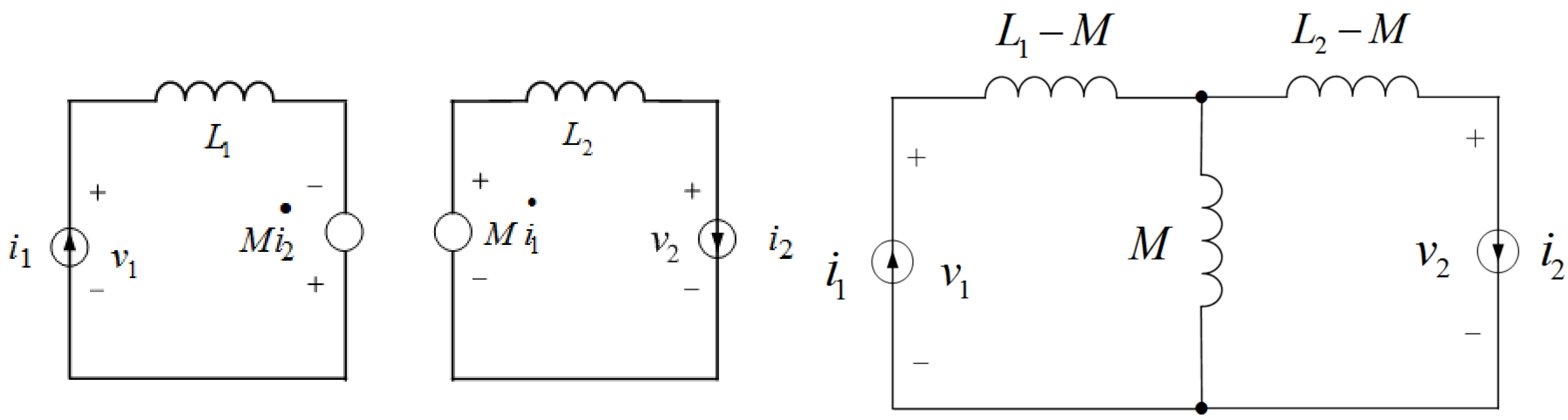


$$v_1 = N_1(\dot{\phi}_{11} + \dot{\phi}_{12} - \dot{\phi}_{21}) = L_{l1}\dot{i}_1 + (L_{12}\dot{i}_1 - L_{21}\dot{i}_2 / n)$$

$$v_2 = -N_2(\dot{\phi}_{22} + \dot{\phi}_{21} - \dot{\phi}_{12}) = -L_{l2}\dot{i}_2 - (L_{21}\dot{i}_2 - nL_{12}\dot{i}_1)$$

Principle of Inductive Power Transfer (IPT)

❑ M-model & T-model

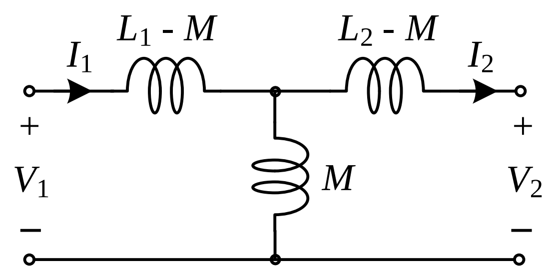


$$v_1 = L_{l1}\dot{i}_1 + L_m(\dot{i}_1 - n\dot{i}_2) = (L_{l1} + L_m)\dot{i}_1 - nL_m\dot{i}_2 \equiv L_1\dot{i}_1 - M\dot{i}_2$$

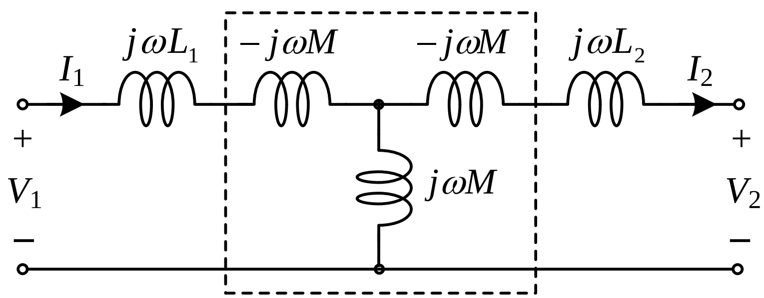
$$v_2 = (nL_m\dot{i}_1 - n^2L_m\dot{i}_2) - L_{l2}\dot{i}_2 = nL_m\dot{i}_1 - (n^2L_m + L_{l2})\dot{i}_2 \equiv M\dot{i}_1 - L_2\dot{i}_2$$

Static Gyrator Model for the Steady State IPT

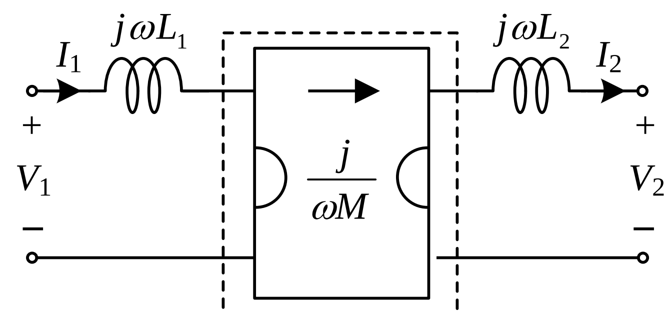
- Static G-Model: $I_1 = \dfrac{-1}{jX_0} \times V_2 \qquad I_2 = \dfrac{1}{jX_0} \times V_1$



(a) Equivalent T-model of IPT



(b) Equivalent circuit of (a)

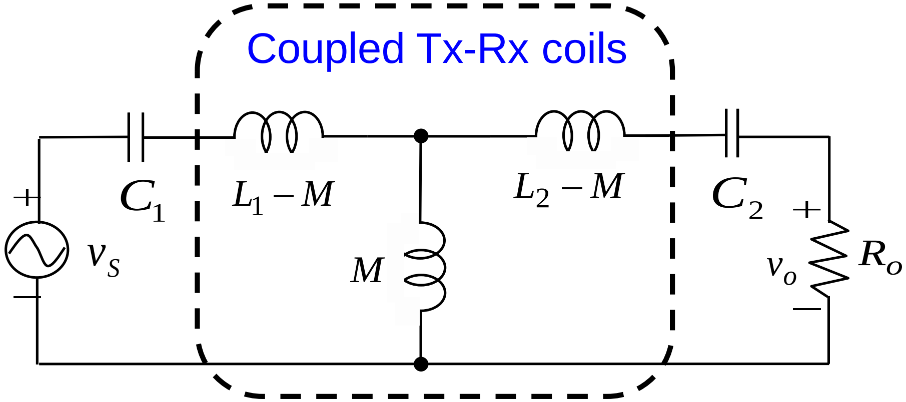


(c) Static gyrator-model

[34] Y. H. Sohn, B. H. Choi, G. -H. Cho and C. T. Rim, "Gyrator-Based Analysis of Resonant Circuits in Inductive Power Transfer Systems," IEEE Transactions on Power Electronics, vol. 31, no. 10, pp. 6824-6843, Oct. 2016.

Static Gyrator Model for the Steady State IPT

❑ **Voltage source S-S IPT example (tuned in the steady state)**

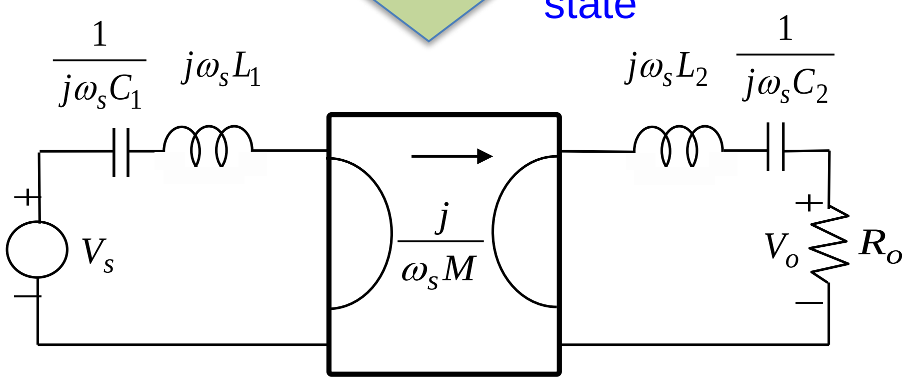


$$V_o = I_o R_o = \frac{jV_s R_o}{\omega_s M}$$

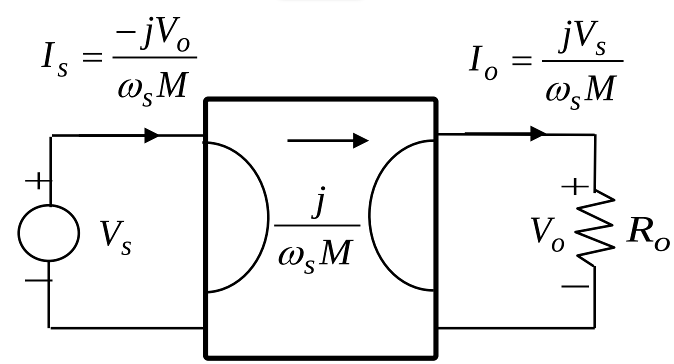
$$\Rightarrow I_s = \frac{-jV_o}{\omega_s M} = \frac{-j}{\omega_s M}\frac{jV_s R_o}{\omega_s M} = \frac{V_s R_o}{(\omega_s M)^2}$$

$$\Rightarrow P_s = P_o = V_s I_s = \frac{V_s^2 R_o}{(\omega_s M)^2} = \frac{V_s^2}{R_{eff}}, \quad \because R_{eff} \equiv \frac{(\omega_s M)^2}{R_o}$$

Steady state



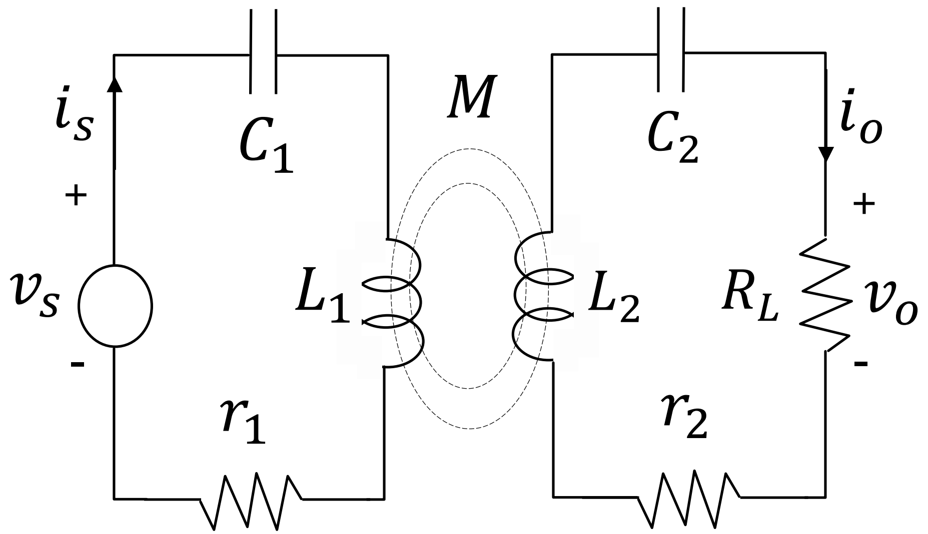
Tuned case



II. Dynamic Phasor-based Gyrator Model of IPT

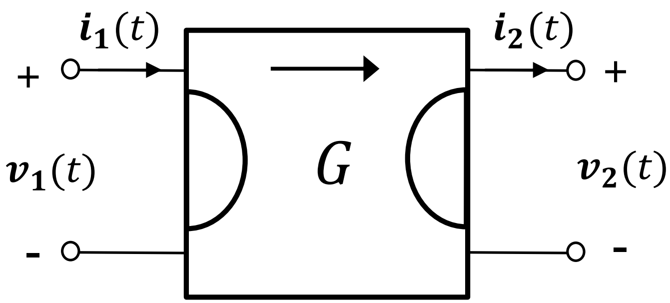
Dynamic Gyrator Model for the Transient State IPT

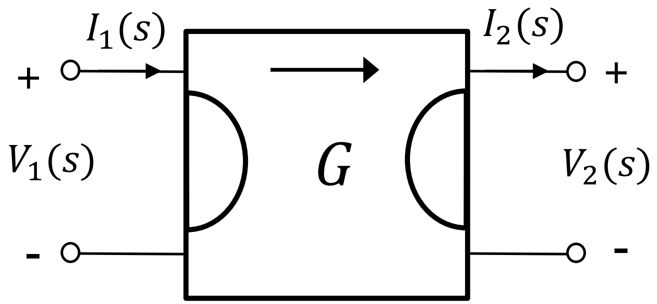
- Proposed S-S IPT with internal resistances



Dynamic Gyrator Model for the Transient State IPT

- Proposed dynamic gyrators (time domain vs. frequency domain)





$$\boldsymbol{i}_2(t) = \boldsymbol{G}\boldsymbol{v}_1(t)$$

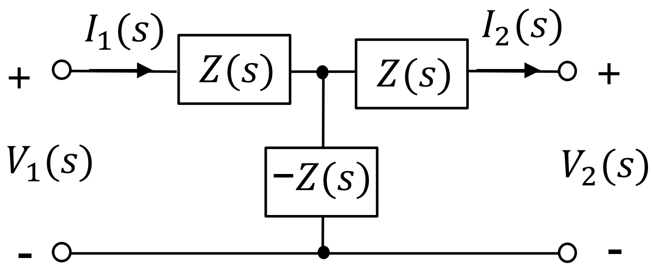
$$\boldsymbol{i}_1(t) = \boldsymbol{G}^*\boldsymbol{v}_2(t)$$

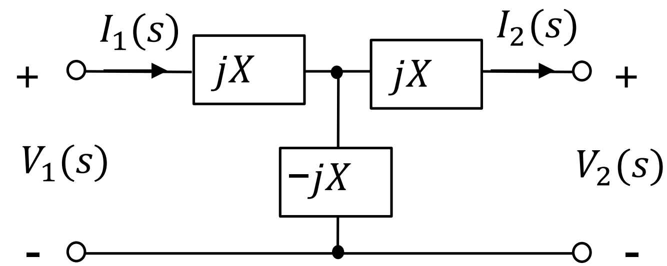
$$I_2(s) = \boldsymbol{G}V_1(s)$$

$$I_1(s) = \boldsymbol{G}^*V_2(s)$$

Dynamic Gyrator Model for the Transient State IPT

Dynamic T-Model: Conditions for Imaginary gyrator





$$V_1(s) = -Z(s)\{I_1(s) - I_2(s)\} + Z(s)I_1(s) = Z(s)I_2(s)$$

$$V_2(s) = -Z(s)\{I_1(s) - I_2(s)\} + Z(s)\{-I_2(s)\} = Z(s)I_1(s)$$

+

$$I_2(s) = \boldsymbol{G}V_1(s)$$

$$I_1(s) = \boldsymbol{G}^*V_2(s)$$

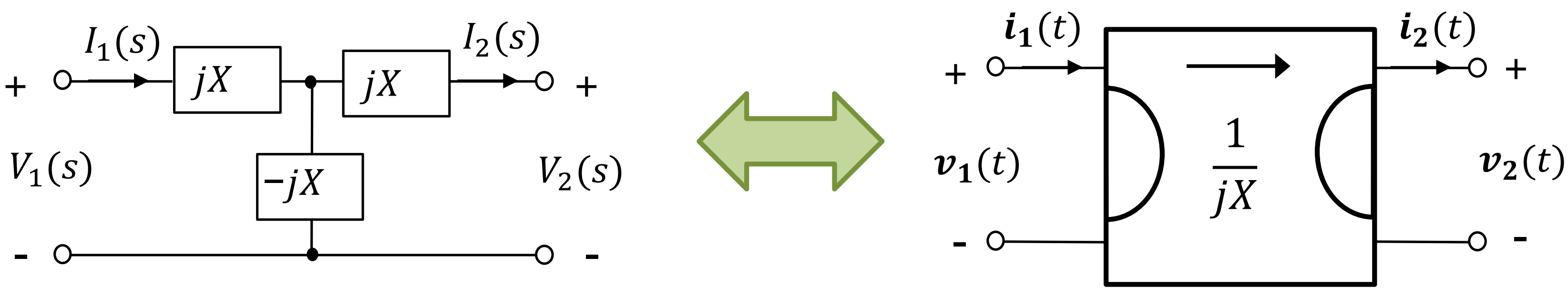
$$\boldsymbol{G} = 1/Z(s), \quad \boldsymbol{G}^* = 1/Z^*(s) = -1/Z(s)$$

$$Z^*(s) = -Z(s) \;\rightarrow\; Re\{Z(s)\} = 0 \;\rightarrow\; Z(s) = jX$$

Z*s=-Zs
⇔ReZs=0

Dynamic Gyrator Model for the Transient State IPT

❑ **Proposed dynamic gyrator valid for complex time domain**

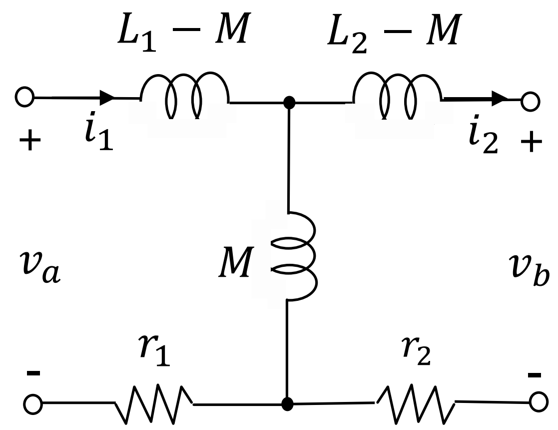


$$I_2(s) = \boldsymbol{G}V_1(s) = V_1(s)/jX$$

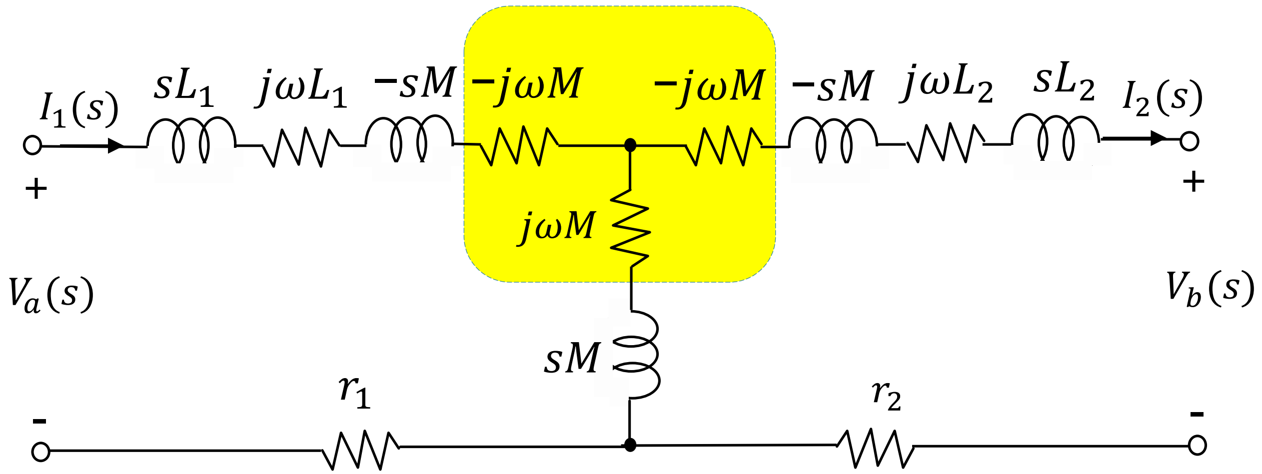
$$I_1(s) = \boldsymbol{G}^*V_2(s) = -V_2(s)/jX$$

Dynamic Phasor Circuit of an IPT

- T-Model in time domain

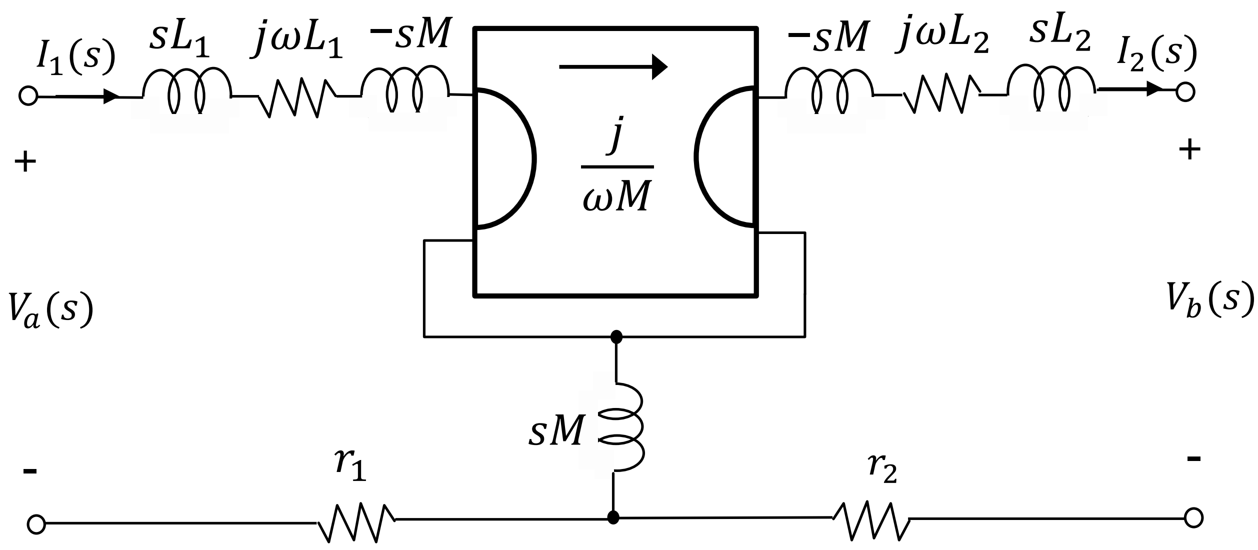


- Dynamic phasor circuit in s-domain



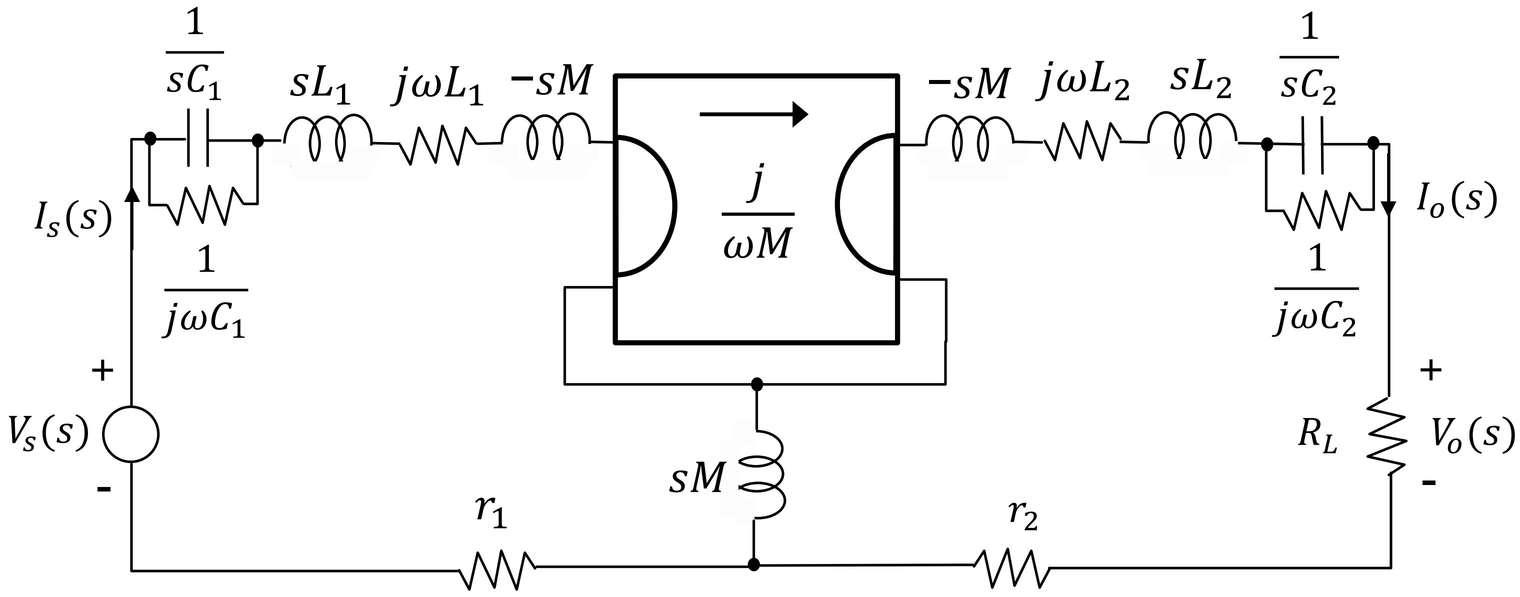
Dynamic Phasor Circuit of an IPT

- Replacement with a dynamic gyrator in the dynamic phasor circuit



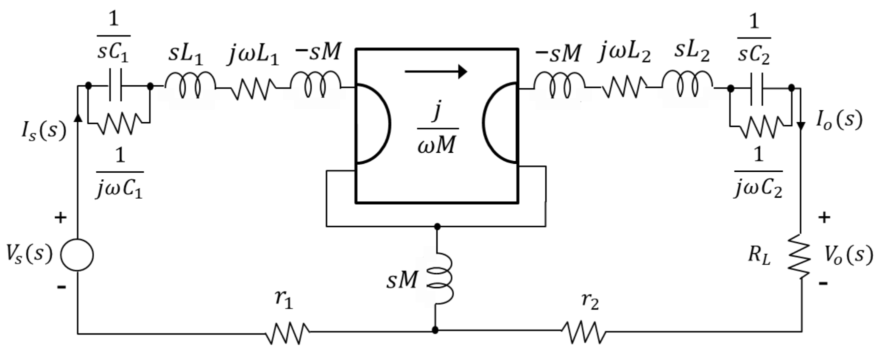
Dynamic Gyrator-based IPT Circuit Analysis

- The dynamic phasor circuit of S-S IPT with a dynamic gyrator



Dynamic Gyrator-based IPT Circuit Analysis

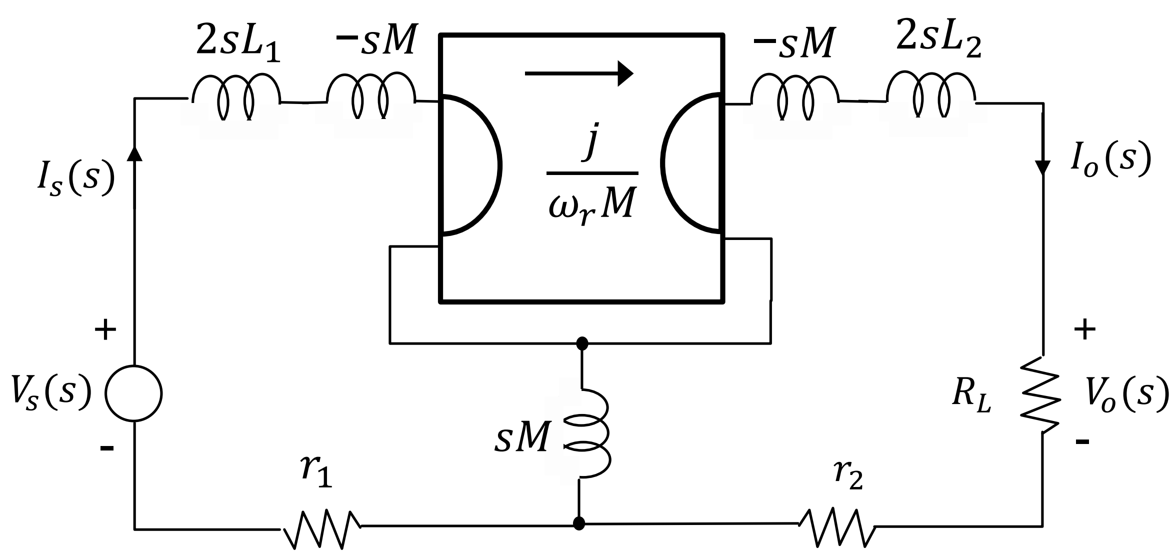
- **A simplified dynamic phasor circuit in tuned case ($\omega = \omega_r$)**



$$Z_s(s) \equiv \frac{1}{sC_1 + j\omega_r C_1} + sL_1 + j\omega_r L_1 = \frac{1}{j\omega_r C_1\left(1+\frac{s}{j\omega_r}\right)} + sL_1 + j\omega_r L_1 \cong$$

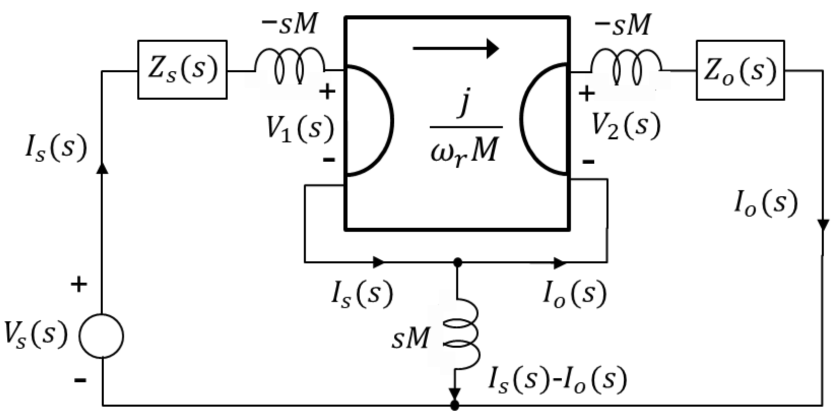
$$\frac{1}{j\omega_r C_1}\left(1 - \frac{s}{j\omega_r}\right) + sL_1 + j\omega_r L_1 = = \frac{1}{j\omega_r C_1} + j\omega_r L_1 + \frac{s}{\omega_r^2 C_1} + sL_1 = 2sL_1,$$

$$\because |s| \ll \omega_r$$



Dynamic Gyrator-based IPT Circuit Analysis

❑ Further simplified approximated circuit, eliminating s-terms

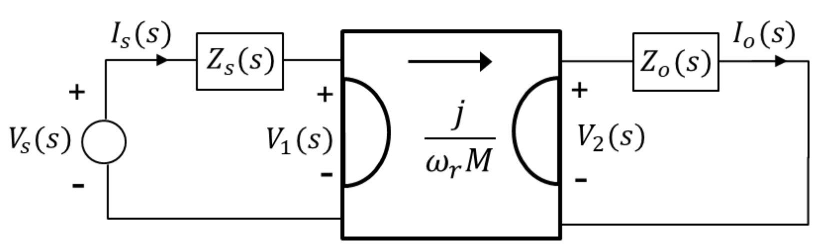


$$V_s(s) = Z_s(s)I_s(s) - sMI_s(s) + sM\{I_s(s) - I_o(s)\} + V_1(s)$$

$$= Z_s(s)I_s(s) - sMI_o(s) + V_1(s) = Z_s(s)I_s(s) + \frac{sMV_1(s)}{j\omega_r M} + V_1(s)$$

$$= Z_s(s)I_s(s) + V_1(s)\left(1 + \frac{s}{j\omega_r}\right) \cong Z_s(s)I_s(s) + V_1(s)\,,\ |s| \ll \omega_r$$

$|s| \ll \omega_r$



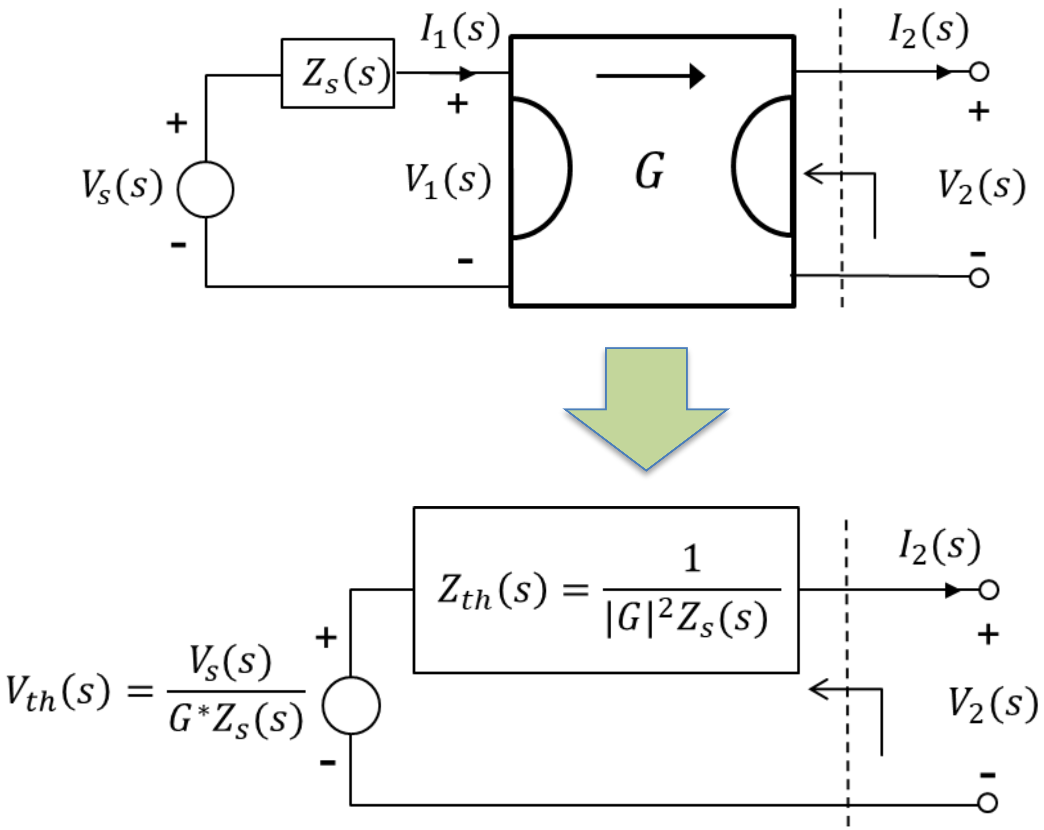
$$0 = Z_o(s)I_o(s) - sMI_o(s) + sM\{I_o(s) - I_s(s)\} - V_2(s)$$

$$= Z_o(s)I_o(s) - sMI_s(s) - V_2(s) = Z_o(s)I_o(s) - \frac{sMV_2(s)}{j\omega_r M} - V_2(s)$$

$$= Z_o(s)I_o(s) - V_2(s)\left(1 + \frac{s}{j\omega_r}\right) \cong Z_o(s)\ \ I_o(s) - V_2(s)\,,\ |s| \ll \omega_r$$

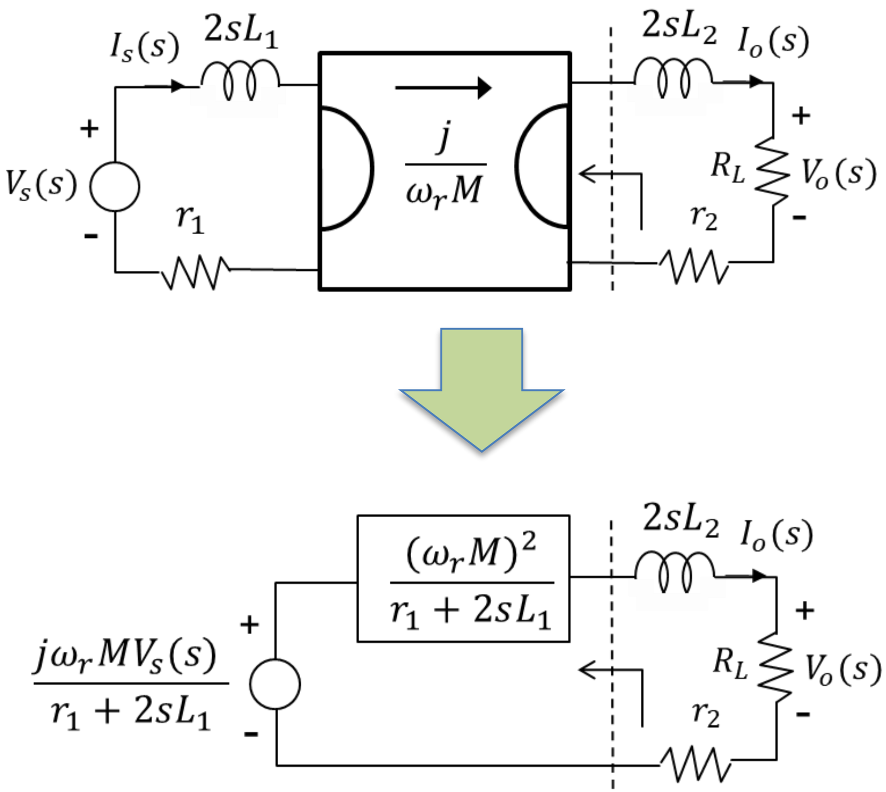
Dynamic Gyrator-based IPT Circuit Analysis

- Elimination of a dynamic gyrator from load side



Dynamic Gyrator-based IPT Circuit Analysis

- **Thevenin equivalent circuit of a dynamic gyrator**



$$G_V(s) \equiv \frac{V_o(s)}{V_s(s)}$$

$$= \frac{j\omega_r M}{r_1 + 2sL_1} \cdot \frac{R_L}{\frac{(\omega_r M)^2}{r_1 + 2sL_1} + r_2 + 2sL_2 + R_L}$$

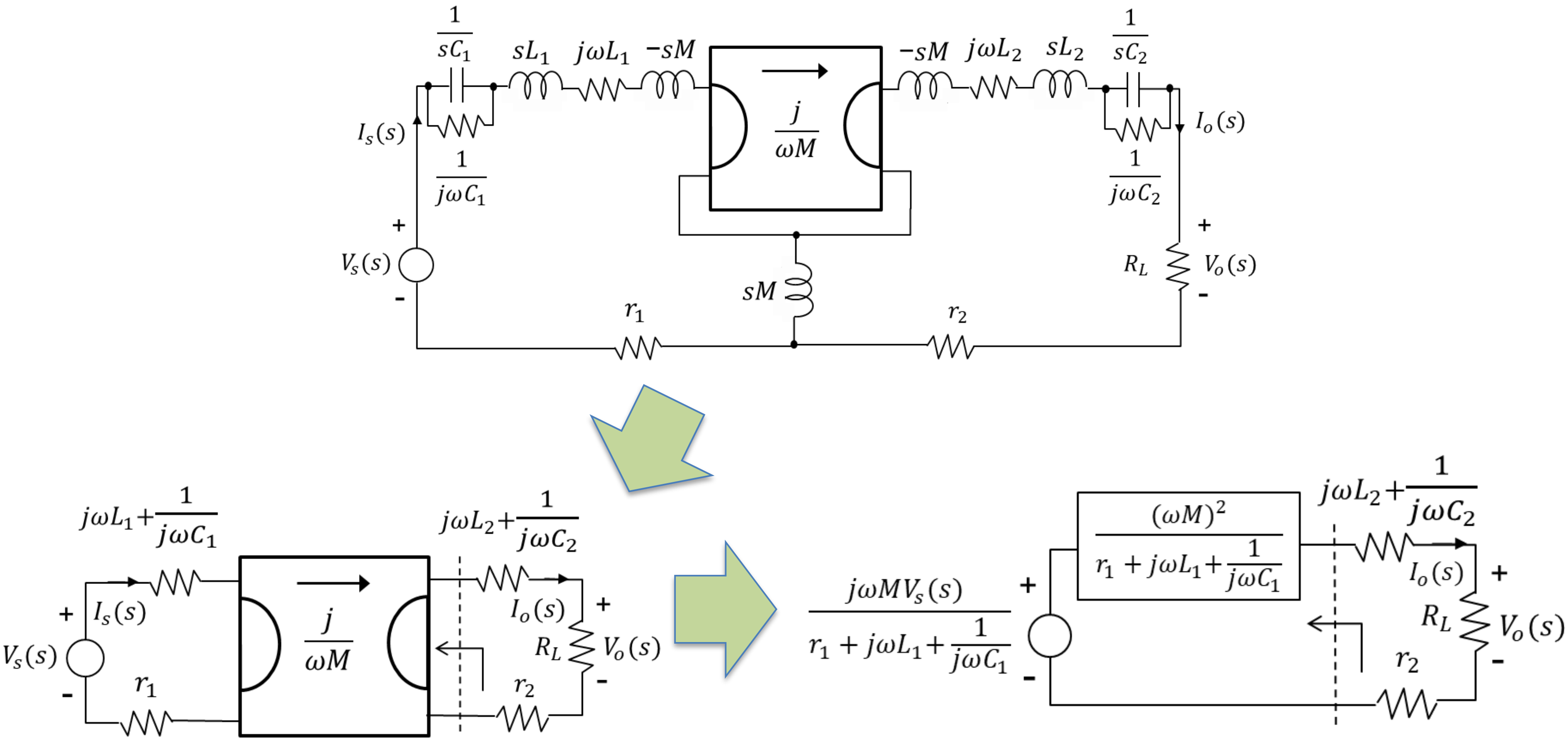
$$= \frac{j\omega_r M R_L}{(\omega_r M)^2 + (r_1 + 2sL_1)(r_2 + R_L + 2sL_2)} =$$

$$\frac{\frac{j\omega_r M R_L}{4L_1 L_2}}{s^2 + \left(\frac{r_1}{2L_1} + \frac{r_2 + R_L}{2L_2}\right) s + \frac{r_1 r_2 + r_1 R_L + (\omega_r M)^2}{4L_1 L_2}}$$

$$\rightarrow \zeta = \frac{\left(\sqrt{\frac{L_2}{L_1}} r_1 + \sqrt{\frac{L_1}{L_2}} R_2\right)}{2\sqrt{r_1 R_2 + (\omega_r M)^2}}$$

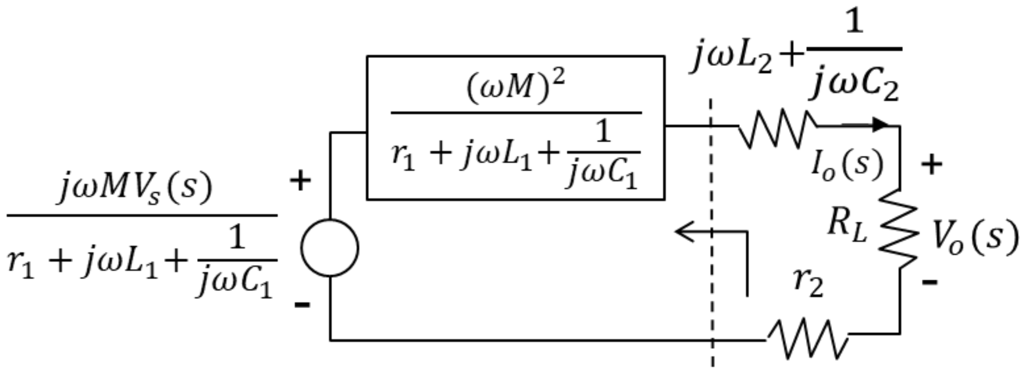
Dynamic Gyrator-based IPT Circuit Analysis

- A simplified dynamic phasor circuit in mistuned case ($\omega \neq \omega_r$)



Dynamic Gyrator-based IPT Circuit Analysis

- A simplified dynamic phasor circuit in mistuned case ($\omega \neq \omega_r$)



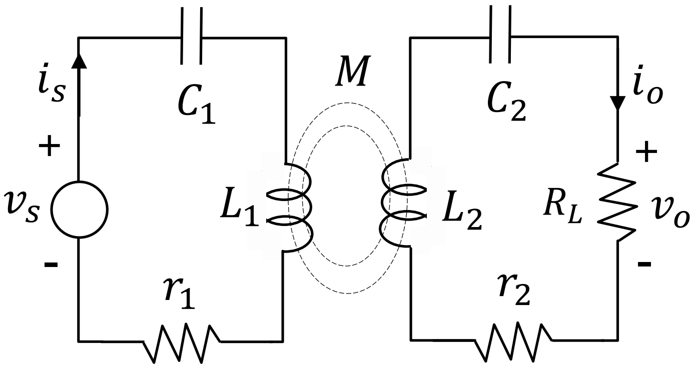
$$G_V(s) \equiv \frac{V_o(s)}{V_s(s)} = G_{V,dc}$$

$$= \frac{j\omega M}{r_1 + j\omega L_1 + \frac{1}{j\omega C_1}} \cdot \frac{R_L}{\frac{(\omega M)^2}{r_1 + j\omega L_1 + \frac{1}{j\omega C_1}} + r_2 + j\omega L_2 + \frac{1}{j\omega C_2} + R_L}$$

$$= \frac{j\omega M\ R_L}{(\omega M)^2 + (r_1 + j\omega L_1 + \frac{1}{j\omega C_1})(r_2 + j\omega L_2 + \frac{1}{j\omega C_2} + R_L)}$$

III. Verifications by Simulation

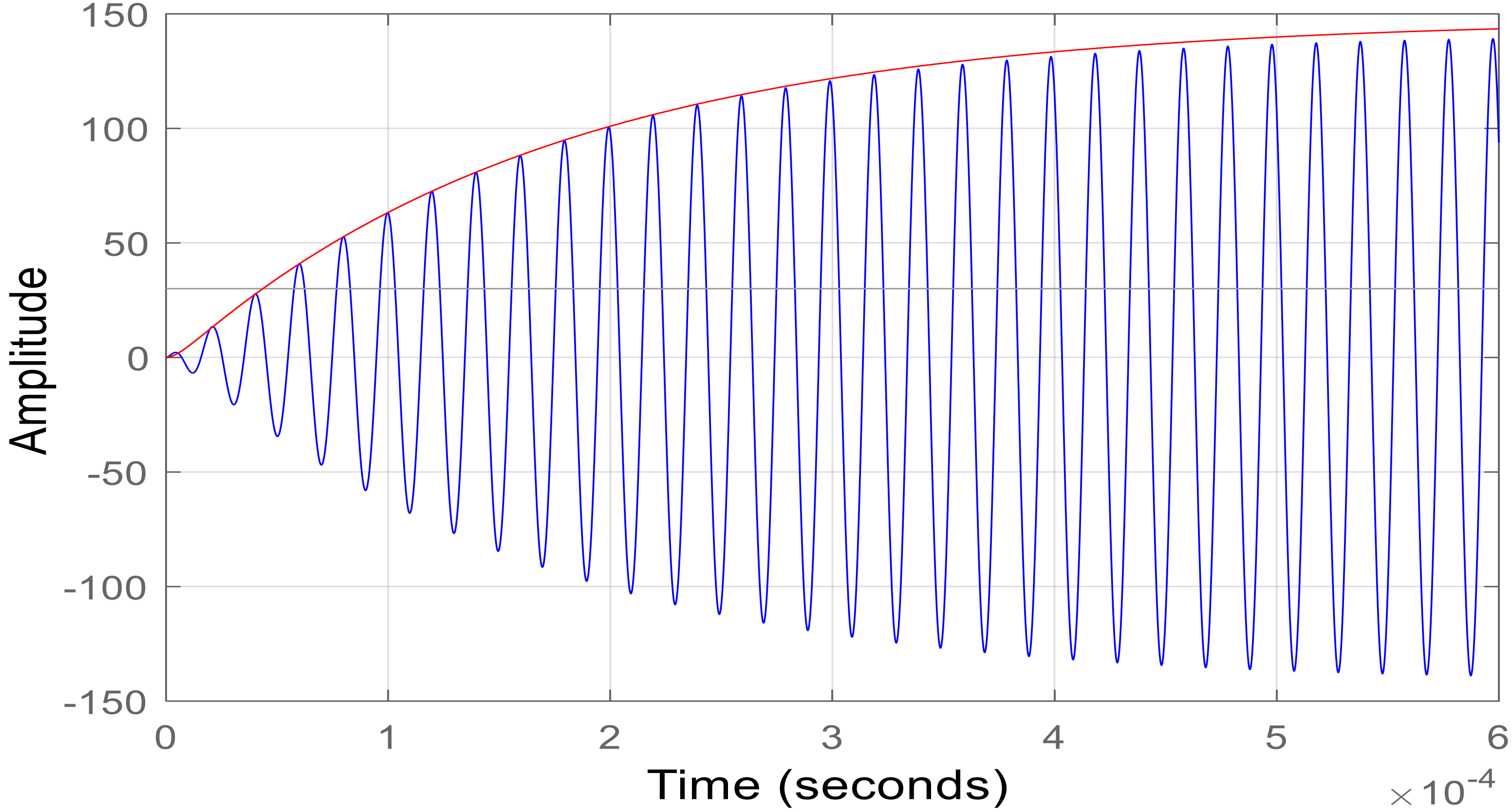
Simulation Conditions for Tuned Case



No.	parameters	Values
1	f_r	50.3 (kHz)
2	L_1	286.3 (μH)
3	L_2	284.8 (μH)
4	M	52.01 (μH)
5	C_1	35.33 (nF)
6	C_2	35.33 (nF)
7	r_1	0.967 (Ω)
8	r_2	1.018 (Ω)
9	R_L	1.29~114.54 (Ω)

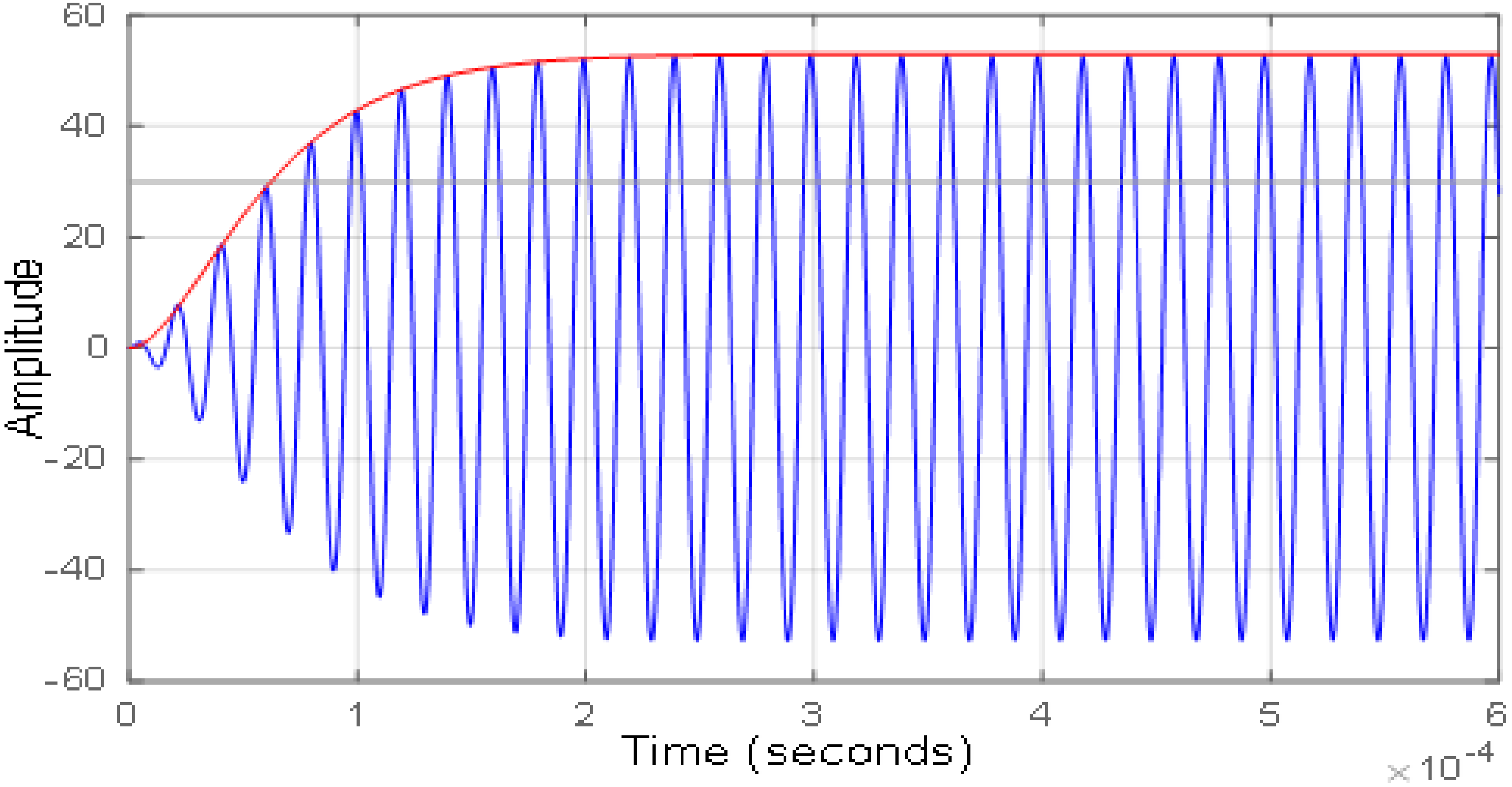
Simulation: ζ =3, R_L = 114.54ohm

$|s| \ll \omega_r$



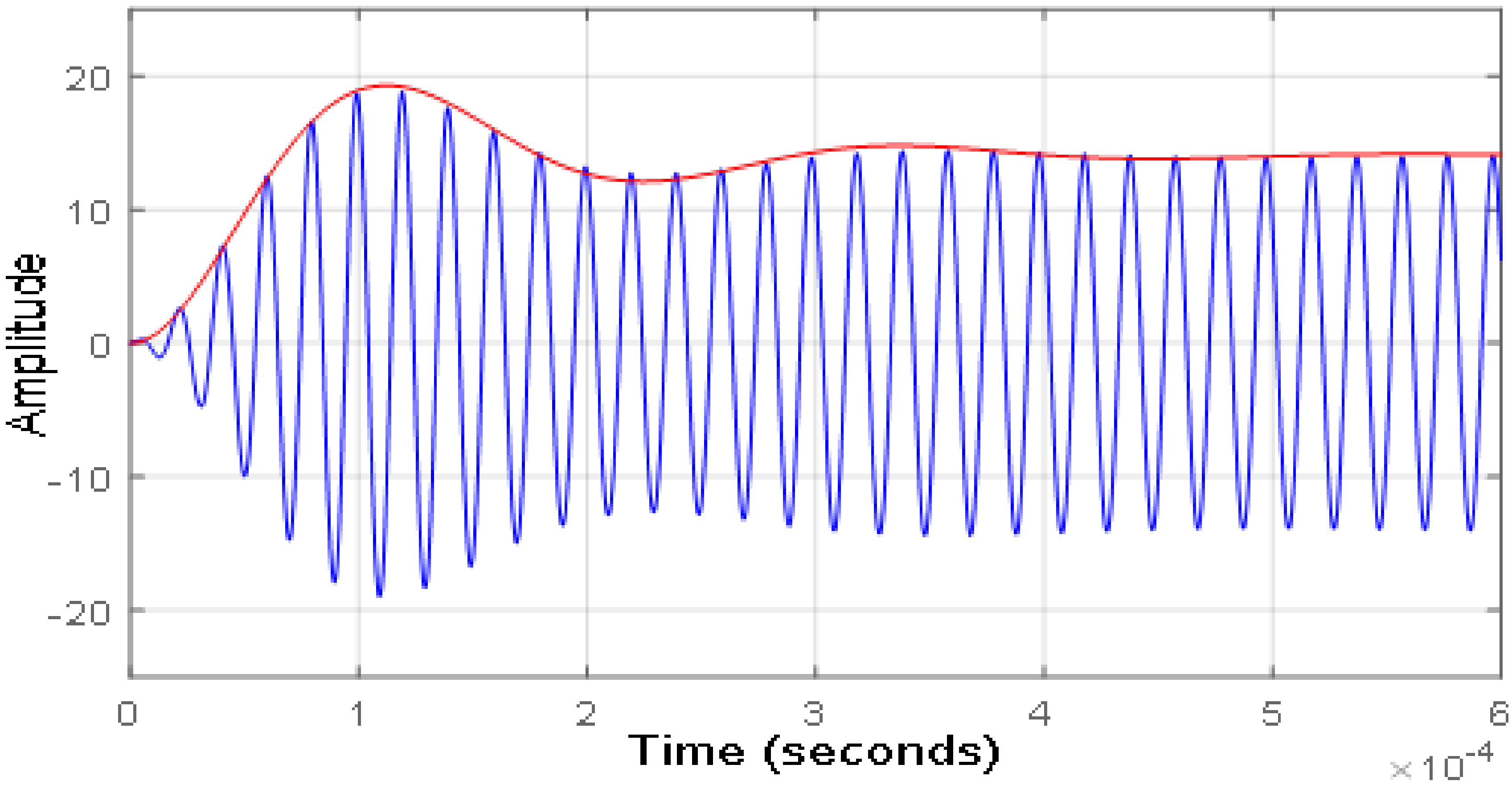
Simulation: ζ =1, R_L = 32.55ohm

$|s| \ll \omega_r$



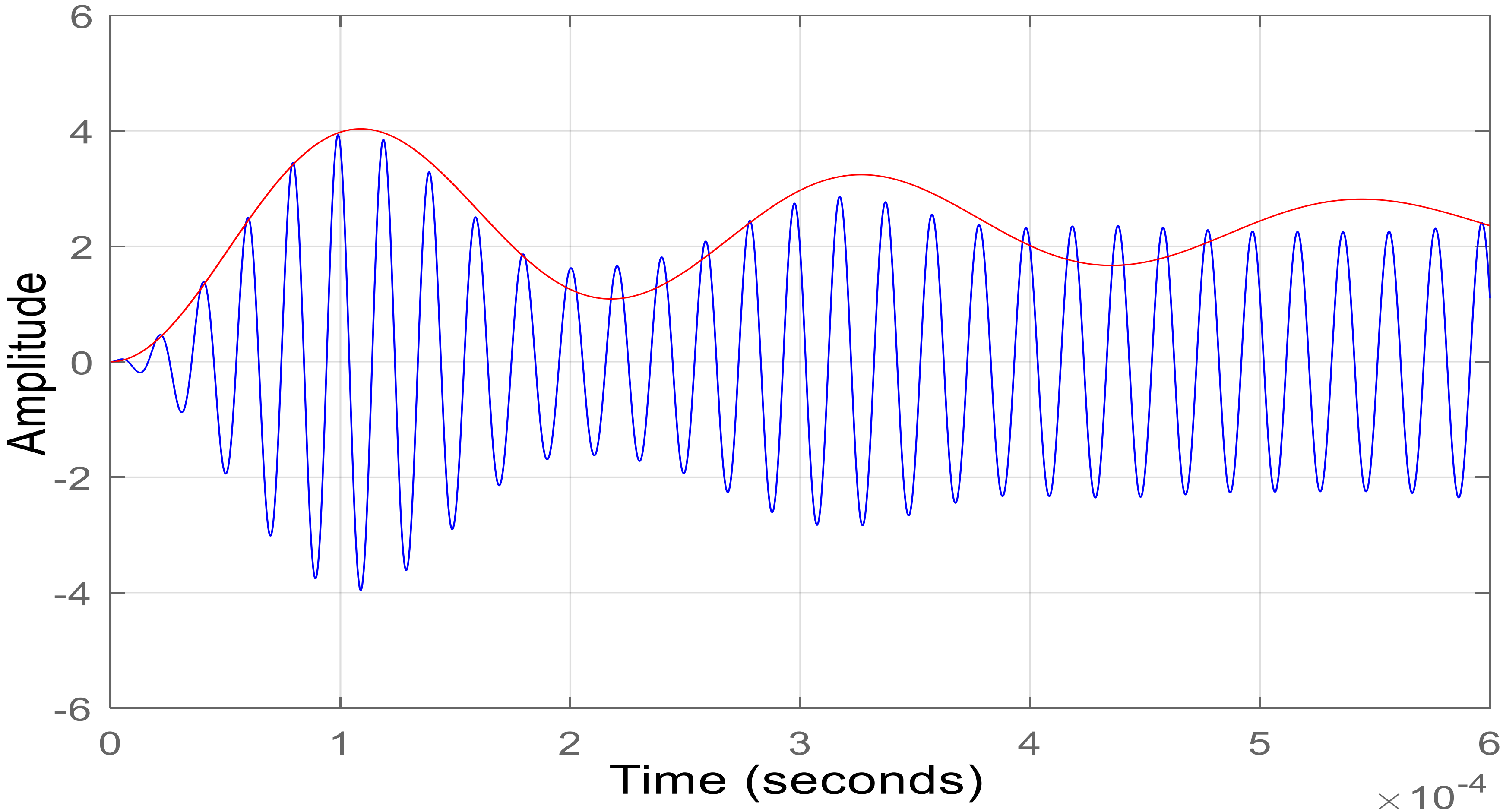
Simulation: ζ =0.3, R_L = 7.95 ohm

$|s| \ll \omega_r$



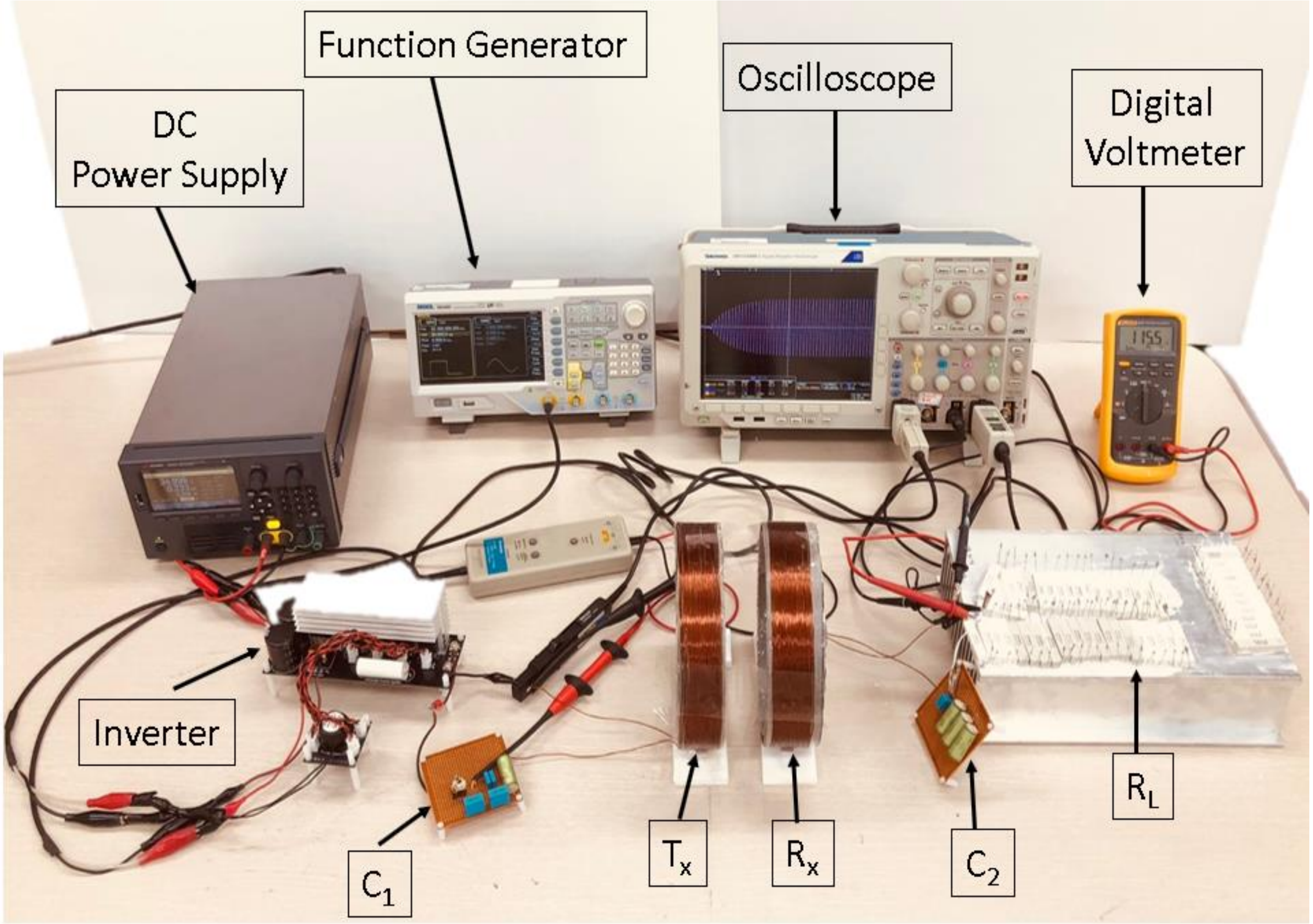
Simulation: ζ =0.1, R_L = 1.29 ohm

$$|s| \ll \omega_r?$$

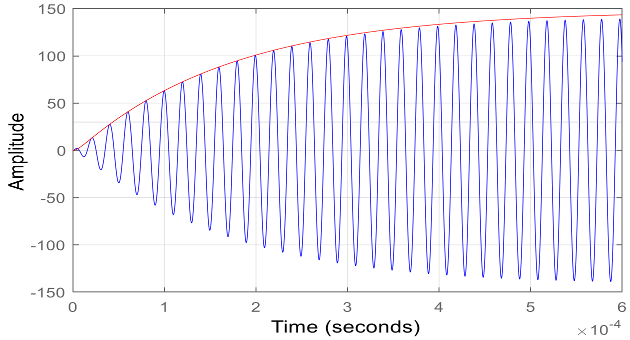


IV. Experimental Verifications

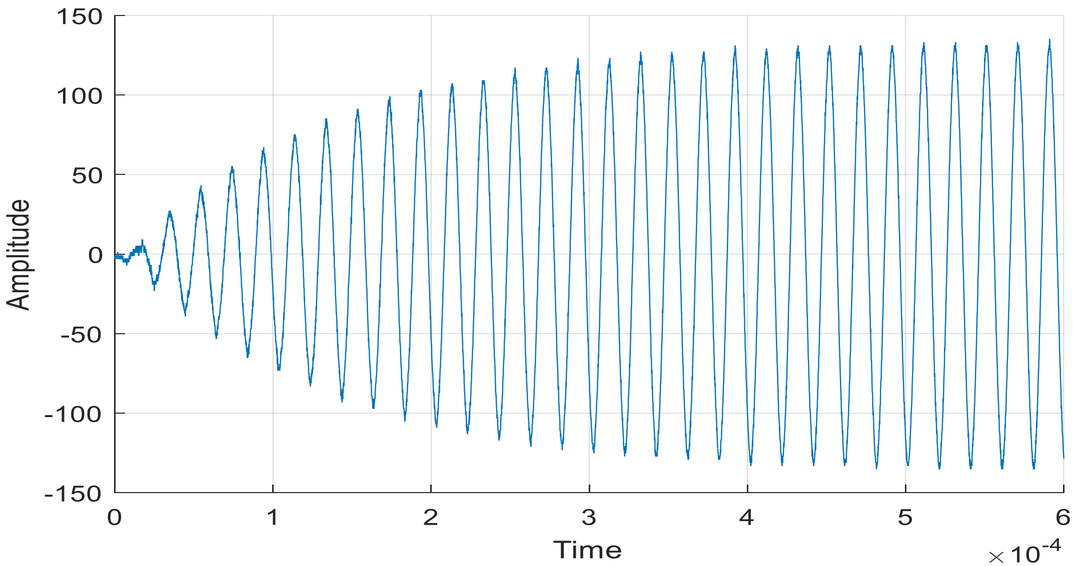
Experimental Kit



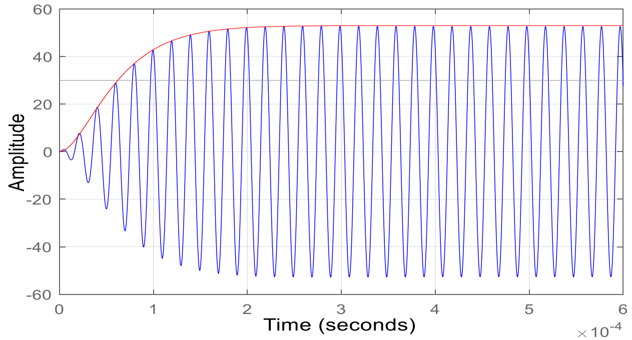
Experiment: ζ =3, R_L = 114.54ohm



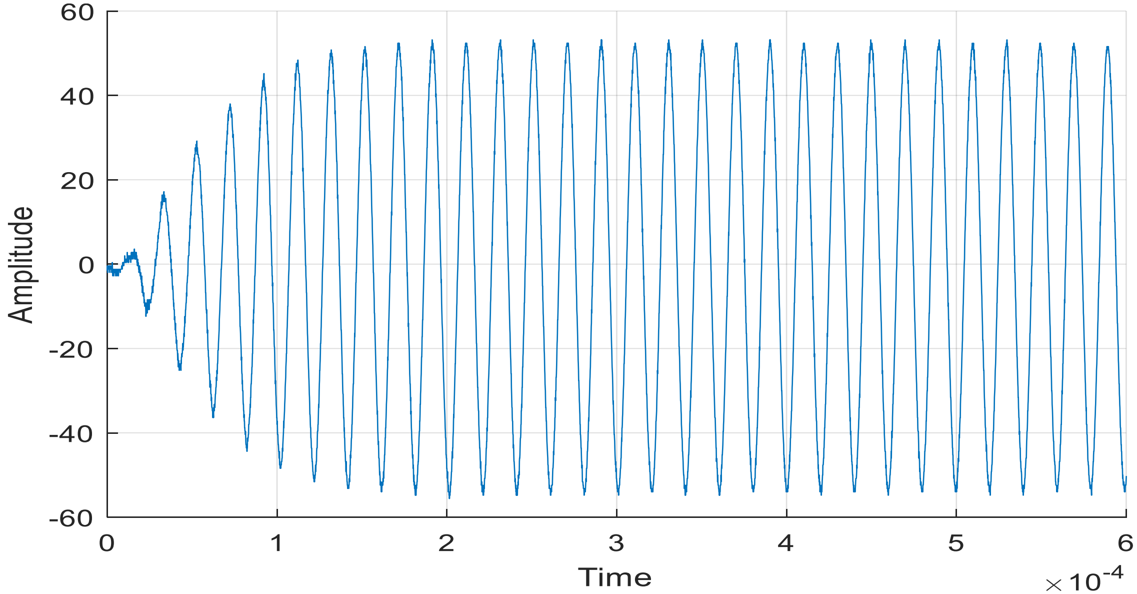
$|s| \ll \omega_r$



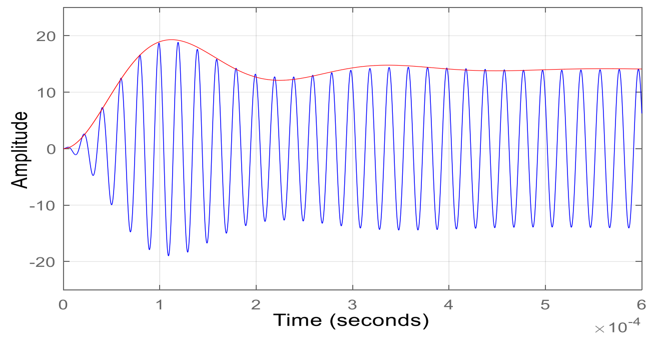
Experiment: ζ =1, R_L = 32.55ohm



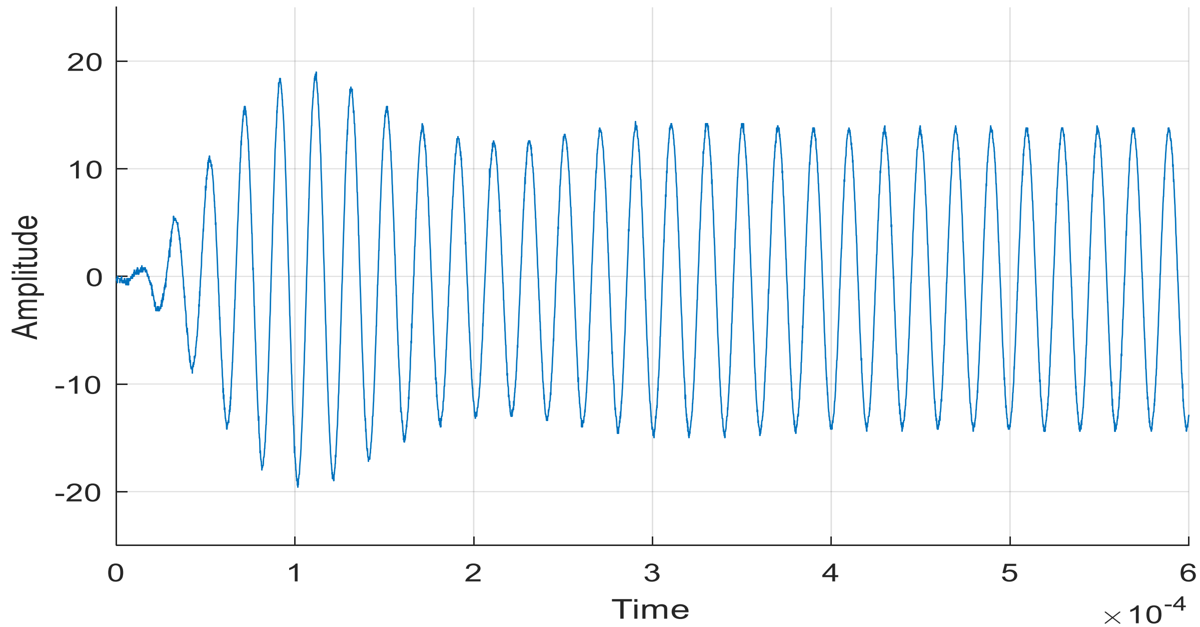
$|s| \ll \omega_r$



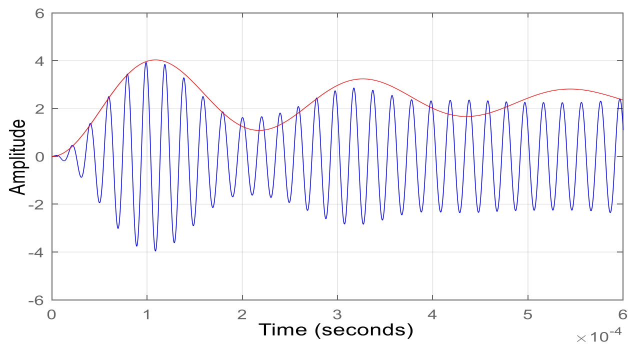
Experiment: ζ =0.3, R_L = 7.95 ohm



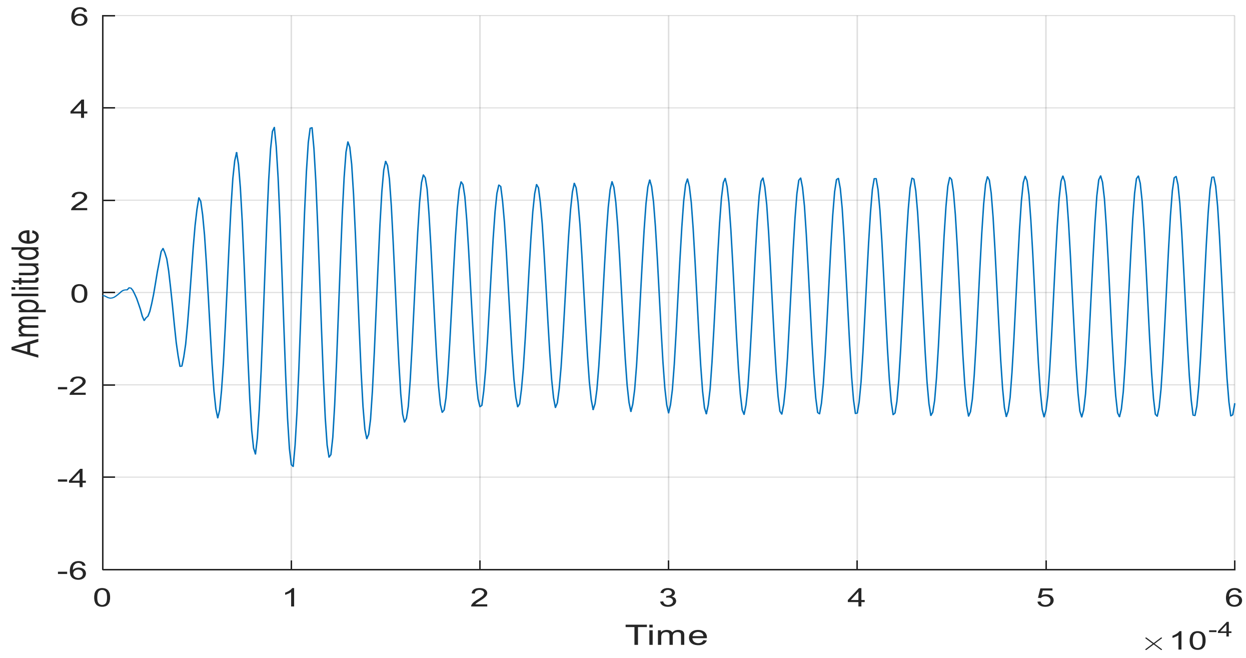
$|s| \ll \omega_r$



Experiment: ζ =0.1, R_L = 1.29 ohm



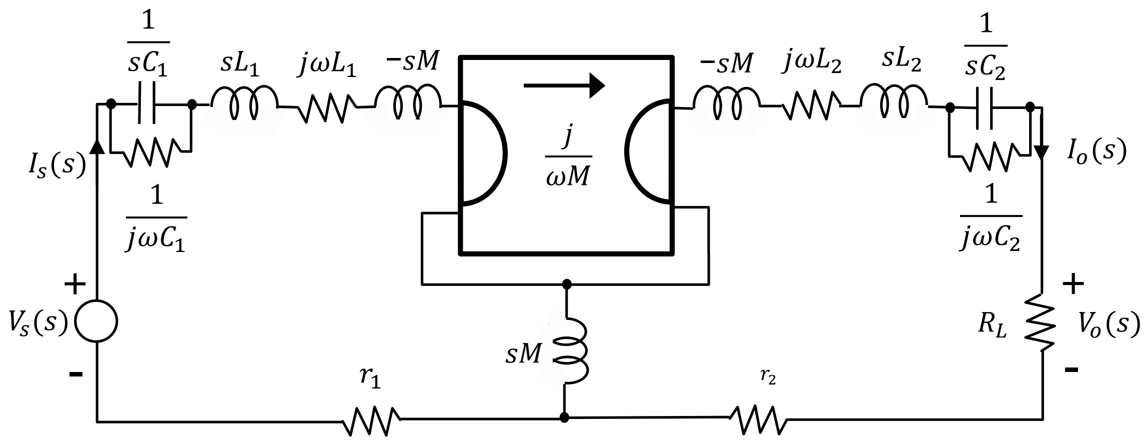
$|s| \ll \omega_r$?



V. Concluding Remark

IPT dynamics is simple & easy now!

- The 5th order IPT system becomes the 2nd order gyrator system



$|s| \ll \omega_r$