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A point projection approach for improving the accuracy of the multilevel B-spline approximation

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ABSTRACT

In this study, we present a method for improving the accuracy of the multilevel B-spline approximation (MBA) method. We combine a point projection method with the MBA method for reducing the approximation error by directly adjusting the control points in the local area. An initial surface is generated by the MBA method, and grid points are produced on the surface. These grid points are projected onto the scattered point set, and the distances between the grid points and the projected points are computed. The control points are then modified based on the distances. The proposed method shows better approximations even with the same number of control points and ensures C^2-continuity. The experimental results with examples verify the validity of the proposed method.

1. Introduction

3D scanning is a method for generating scattered data points that represent the geometric shape of an object. With the improvement in scanning software and hardware technology, 3D scanning is being used in various fields, such as computer graphics, computer-aided design, topographic survey, manufacturing, and medical surgery, for a wide variety of applications. However, there are certain limitations when scanned data points are used in practice. For instance, these points are unstructured in most cases; therefore, it is difficult to derive any useful relations between them. These relations may be necessary for computation of derivatives, or other intrinsic properties. Moreover, a scanner typically generates a large number of points, resulting in long processing times. Thus, new advanced methods are required for analyzing data, such as reconstructing a surface from a reduced number of scattered data points and using this surface for extracting various properties or for data reduction. This approach has been utilized in various applications (Bertram, Tricoche, & Hagen, 2003; Carballido-Gamio & Majumdar, 2011; Lee, Chung, Kim, Lee, & Park, 2005; Seo & Chen, 2009; Wang & Amini, 2011).

Among scattered data fitting methods, the multilevel B-spline approximation (MBA) method (Lee, Wolberg, & Shin, 1997) is widely used in practice. This method creates a surface that interpolates the scattered data points using the least squares approach. If the approximation error is larger than the user defined tolerance, the method increases the number of control points through refining a control net in a hierarchical manner and reduces the approximation error iteratively. However, the method may not yield a satisfactory result at a fixed hierarchy level. This problem is more clearly noticed when sharp changes occur in the geometric shape defined by the point set. Moreover, as the control net is refined to reduce the approximation error, the number of control points increases. Consequently, computation time increases. A great deal of research has been conducted to improve the MBA method. Zhang, Tang, and Li (1998) proposed a method for adaptively finding an area of large error and performing refinement therein. However, this method increased the number of control points for maintaining C^2-continuity. Bertram et al. (2003) proposed an approach that combined adaptive clustering with an approximation by piecewise polynomials. This approach localized the computation of the multilevel control lattice and improved efficiency in terms of computation time. Later, Seo and Chen (2010) suggested an adaptive lattice partitioning method for reducing computation cost. Bracco, Giannelli, and Sestini (2017) suggested a new method for scattered data fitting using a local approximation technique. The hierarchical b-spline can also be implemented based on the subdivision scheme as proposed in Bornemann and Cirak (2013), which relates the basis functions and the coefficients on different levels algebraically.

Minimizing the number of control points is another issue of the MBA method. The accuracy of approximation can be easily improved by refinements, which increase the number of control points by a factor of four. However, it is beneficial to maintain the number of control points as small as possible while the

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accuracy requirement is satisfied because this saves memory and reduces the computation time in the subsequent process.

In this study, a method for improving the accuracy of MBA is proposed. It combines the MBA method (Lee et al., 1997) with the point projection approach. Initially, the MBA method is applied to obtain a surface with control points at one hierarchical level. Subsequently, the grid points on the surface are projected onto the input points. The distances between the surface and the projected points are then computed and applied to the control points.

The paper is structured as follows. The overall process and the detailed steps of the proposed method are presented in Section 2. Results and discussion are presented in Section 3. Section 4 concludes the paper with recommended future work.

2. Overall procedure

The overall process of the proposed method is shown in Fig. 1. 2.5D scattered data points are assumed to be given. MBA is first utilized to form an initial surface for approximating the scattered data. Points are then generated on the surface by creating a grid in the parametric domain and mapping the grid points on the surface. The points on the surface are projected onto the scattered data using a point projection approach. Finally, a B-spline surface is reconstructed by adjusting the control points, considering the distances between the points on the surface and the projected points on the scattered data. A detailed explanation is presented in the following sections.

2.1. Generation of an initial surface

The initial surface for approximating the 2.5D scattered data points is generated using the MBA method. This method is well suited for approximating 2.5D data points by B-splines. The number of control points on the surface is determined by the approximation error. The level of refinement, which determines the number of control points for approximation, is organized in a hierarchical manner. For completeness, a summary of the method is presented in the subsequent sections (Lee et al., 1997).

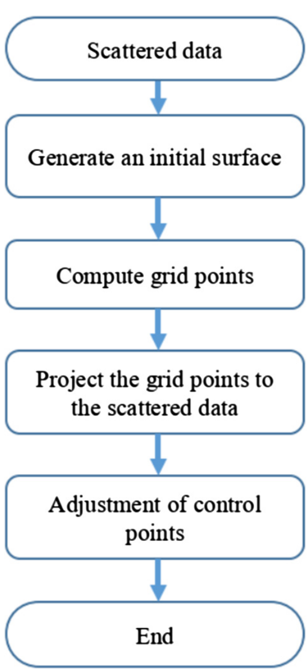


Fig. 1. Flowchart of the proposed method.

2.1.1. B-spline approximation in MBA

We assume that there is a rectangular domain Ω in the *xy*-plane ($\Omega = \{(x, y) \mid 0 \leqslant x \leqslant m,\ 0 \leqslant y \leqslant n\}$) and a control lattice Φ with $(m+3) \times (n+3)$ control points located at (i,j) for $i = -1, 0, 1, \ldots, m+1$ and $j = -1, 0, \ldots, n+1$. The control lattice Φ is defined large enough to cover the domain Ω as shown in Fig. 2. Then, the approximation function f is defined as

$$f(x,y) = \sum_{k=0}^{3}\sum_{l=0}^{3} B_k(s)B_l(t)\phi_{(i+k)(j+l)}, \tag{1}$$

where $i = \lfloor x \rfloor - 1, j = \lfloor y \rfloor - 1, s = x - \lfloor x \rfloor$ and $t = y - \lfloor y \rfloor$. B_k and B_l are the uniform cubic B-spline basis functions.

When there is a point $\mathbf{p} = (x_c, y_c, z_c)$, the control points ϕ_{kl} can be determined in the least squares sense to generate a surface interpolating the point as follows (Lee et al., 1997):

$$\phi_{kl} = \frac{B_k(s)B_l(t)z_c}{\sum_{a=0}^{3}\sum_{b=0}^{3}(B_a(s)B_b(t))^2}. \tag{2}$$

If more than two points $\mathbf{p}^q = (x_c^q, y_c^q, z_c^q), q = 1, 2, 3, \cdots, n^q$ are near each other, Eq. (2) yields a different control point value ϕ_c^q for each point $\mathbf{p}^q$ (Lee et al., 1997).

$$\phi_c^q = \frac{B_k(s)B_l(t)z_c^q}{\sum_{a=0}^{3}\sum_{b=0}^{3}(B_a(s)B_b(t))^2}. \tag{3}$$

Here, $k = (i+1) - \lfloor x_c \rfloor, l = (j+1) - \lfloor x_c \rfloor, s = x_c - \lfloor x_c \rfloor,$ and $t = y_c - \lfloor y_c \rfloor$. This implies that there are multiple candidate control points ϕ_c^q for determining ϕ_{ij}.

The optimal control point ϕ_{ij} that minimizes the approximation error is determined by (Lee et al., 1997)

$$\phi_{ij} = \frac{\sum_{q=1}^{n^q}\left[\sum_{k=0}^{3}\sum_{l=0}^{3}(B_k(s)B_l(t))^2\phi_c^q\right]}{\sum_{q=1}^{n^q}\left[\sum_{k=0}^{3}\sum_{l=0}^{3}(B_k(s)B_l(t))^2\right]}, \tag{4}$$

where $i = \lfloor x_c \rfloor - 1, j = \lfloor y_c \rfloor - 1, s = x_c - \lfloor x_c \rfloor$ and $t = y_c - \lfloor y_c \rfloor$.

When there is no point $\mathbf{p} = (x_c, y_c, z_c)$, a zero value is assigned to ϕ_{ij}.

2.1.2. Multilevel B-spline approximation

The computation process in the previous section is applied to the multilevel B-spline approximation scheme. We consider a control lattice ϕ_0 of size $(m+3) \times (n+3)$. Then, the control lattice ϕ_1 after the refinement is of size $(2m+3) \times (2n+3)$. The space of the

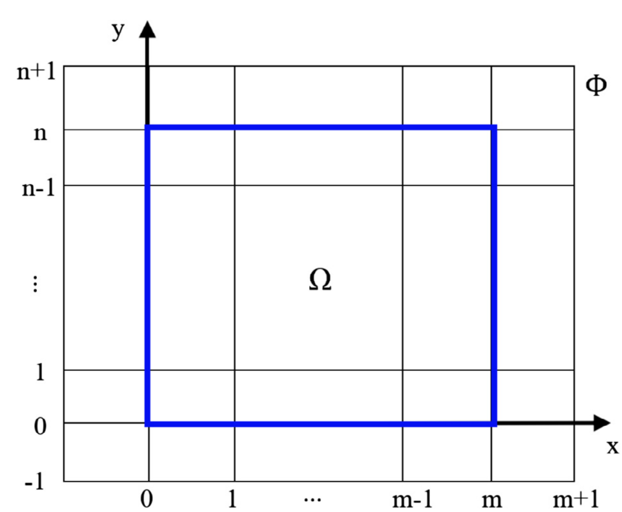


Fig. 2. Rectangular domain Ω (thick blue lines) and control lattice Φ (thin black lines).

control lattice is thereby halved, and the control lattice becomes finer.

Fig. 3 shows the diagram of the multilevel B-splines approximation. f_0,f_1 and f_2 refer to the surface functions. ϕ_0, ϕ_1 and ϕ_h refer to the control lattices. Three levels of control lattices are considered: ϕ_0 with a set of 8×8 control points, ϕ_1 with a set of 13×13 control points, and ϕ_2 with a set of 18×18 control points. ϕ_0 is calculated and then the surface function f_0 is generated using ϕ_0. The error between the real and the estimated values is calculated and used for generating the next control lattice. Likewise, f_1 is generated again using the ϕ_1 control lattice, and an error is incurred by f_1. This error is used for the determination of ϕ_2. This process of refinement and computation of control points is repeated until the approximation error becomes less than a user defined tolerance.

2.2. Generation of grid points

Once the initial surface is generated, grid points are extracted from the surface. The x and y coordinates of the grid points are selected such that they match those of the control points of the control lattice. Then, the grid points are overlaid on the control lattice as shown in Fig. 4.

The rectangular domain Ω is discretized uniformly to produce $m+1$ and $n+1$ points in the x and y directions. A uniform cubic B-spline of degree d is then created using Ω for the knot vectors $\mathbf{T}_s$ and $\mathbf{T}_t$ as follows:

$$\mathbf{T}_s = \{s_i | s_i = i/(m+3+d), \quad i = 0,1,2,\cdots,m+3+d\},$$
$$\mathbf{T}_t = \{t_j | t_j = j/(n+3+d), \quad j = 0,1,2,\cdots,n+3+d\}.$$

Once the surface is defined, the parameters for the grid are obtained using the knot values

$$p_{ij} = (u_i, v_j),$$
$$u_i = s_{i+3}, \quad i = 0,1,\cdots,m,$$
$$v_j = t_{j+3}, \quad j = 0,1,\cdots,n.$$

Next, the surface is evaluated at p_{ij} to produce the grid points, which are illustrated in Fig. 4.

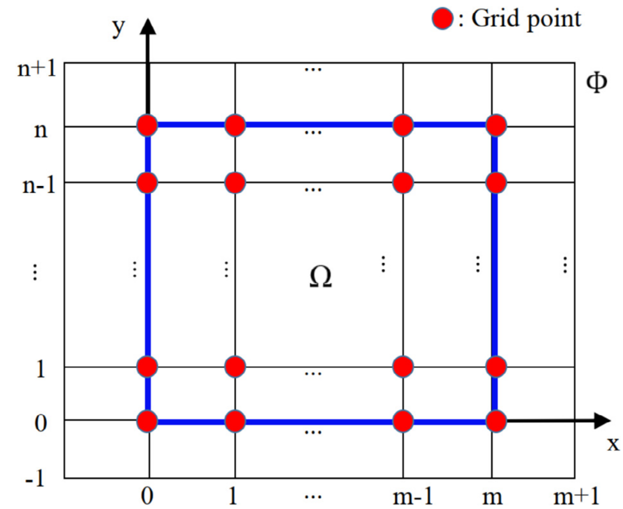


Fig. 4. Discretized points in the domain Ω and the control lattice Φ laid over the domain.

2.3. Projection of the grid points to the scattered data

After the grid points are generated, they are projected onto the scattered data as shown in Fig. 5. To this end, the point projection approach is utilized. The steps of point projection are summarized as follows (Azariadis & Sapidis, 2005). Let us assume that there is a point $\mathbf{p} = (x, y, z)$ to be projected onto the n points $\mathbf{p}_d = (x_d, y_d, z_d), d = 0, 1, \cdots n-1$. Then, the sum of the weighted squared distances with a weight factor a_d is defined as

$$\begin{aligned} E(\mathbf{p}) &= \sum_{d=0}^{n-1} a_d \|\mathbf{p} - \mathbf{p}_d\|^2 \\ &= \sum_{d=0}^{n-1} a_d [(x-x_d)^2 + (y-y_d)^2 + (z-z_d)^2]. \end{aligned} \tag{5}$$

Eq. (5) can be efficiently computed using a five dimensional vector $\mathbf{c}$ (Erikson & Manocha, 1999).

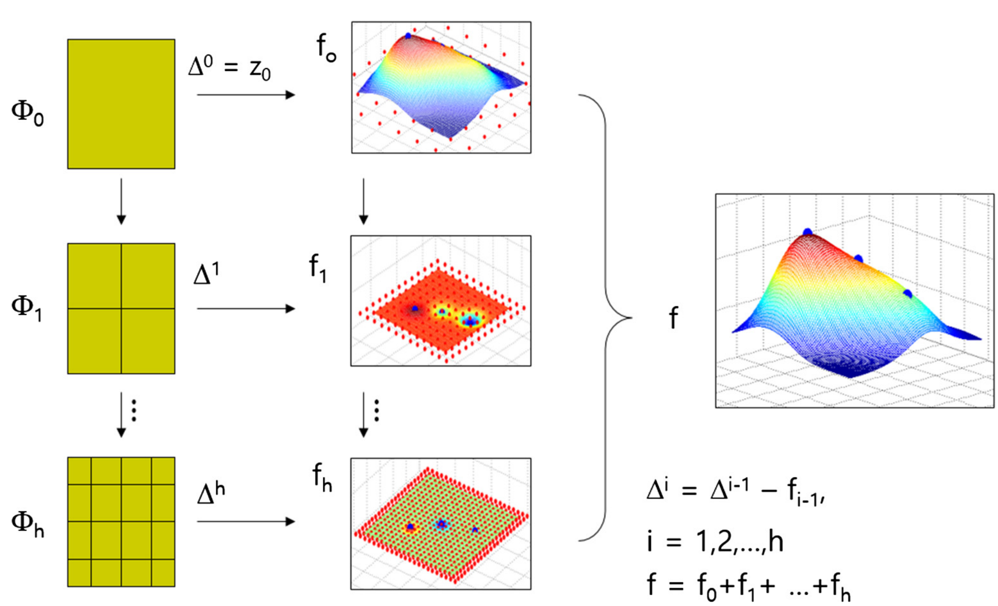


Fig. 3. Process of multilevel B-spline approximation.

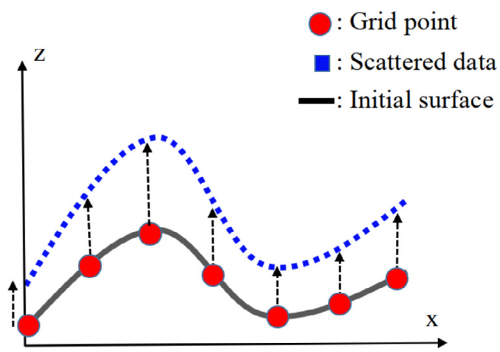


Fig. 5. Point projection from an initial surface to the scattered data.

$$\begin{aligned}\mathbf{c} =&(c_0, c_1, c_2, c_3, c_4)\\ c_0 =& \sum_{d=0}^{n-1} a_d, \quad c_1 = \sum_{d=0}^{n-1} a_d x_d, \quad c_2 = \sum_{d=0}^{n-1} a_d y_d,\\ c_3 =& \sum_{d=0}^{n-1} a_d z_d, \quad c_4 = \sum_{d=0}^{n-1} a_d ({x_d}^2 + y_d^2 + z_d^2),\end{aligned} \tag{6}$$

$$E(\mathbf{p}) = c_0(x^2 + y^2 + z^2) - 2(c_1 x + c_2 y + c_3 z) + c_4. \tag{7}$$

We assume that $\mathbf{p}^* = (x^*, y^*, z^*)$ is the result of the projection along the direction $\mathbf{n} = (n^x, n^y, n^z)$. Then, $\mathbf{p}^*$ can be defined as

$$\mathbf{p}^* = \mathbf{p}^*(t) = \mathbf{p} + t\mathbf{n} \tag{8}$$

Eq. (7) can be rewritten using Eq. (8) as follows (Erikson & Manocha, 1999):

$$\begin{aligned}E(\mathbf{p}^*(t)) =& c_0((x^*(t))^2 + (y^*(t))^2 + (z^*(t))^2)\\ &- 2(c_1 x^*(t) + c_2 y^*(t) + c_3 z^*(t)) + c_4.\end{aligned} \tag{9}$$

The minimum of Eq. (9) is obtained by determining a root of the derivative of Eq. (9) with respect to t equal to zero as follows (Erikson & Manocha, 1999):

$$\frac{dE(\mathbf{p}^*(t))}{dt} = E'(\mathbf{p}^*(t)) = 0 \ \Rightarrow \ t = \frac{\lambda - \mathbf{pn}}{\|\mathbf{n}\|^2}, \tag{10}$$

$$\lambda = \frac{(c_1 n^x + c_2 n^y + c_3 n^z)}{c_0}, \tag{11}$$

$$\frac{d^2 E(\mathbf{p}^*(t))}{dt^2} = E''(\mathbf{p}^*(t)) = 2c_0\|\mathbf{n}\|^2 > 0. \tag{12}$$

Eq. (12) shows that t in Eq. (10) is a minimum solution for Eq. (9). This t is utilized for projecting the point $\mathbf{p}$ onto the points obtained by Eq. (8). During the projection process, the weight factor a_d plays a key role. Three methods for estimating the weight factors are suggested in Azariadis and Sapidis (2005), Azariadis (2004), Moon, Park, and Ko (2017).

$$a_d = \frac{1}{\|\mathbf{p} - \mathbf{p}_d\|^4}, \quad a_d \in [0, \infty], \tag{13}$$

$$a_d = \frac{1}{1 + \|\mathbf{p} - \mathbf{p}_d\|^2 \|(\mathbf{p}_d - \mathbf{p}) \times \mathbf{n}\|^2}, \quad a_d \in [0, 1]. \tag{14}$$

$$a_d = \frac{1}{\|(\mathbf{p}_d - \mathbf{p}) \times \mathbf{n}\|^4}, \quad a_d \in [0, \infty]. \tag{15}$$

Eq. (13) assigns a large weight to the points near the point to be projected (Azariadis, 2004), and Eq. (14) adds the distance between the projection in direction $\mathbf{n}$ and the points (Azariadis & Sapidis, 2005). These two equations yield acceptable values for applications. However, they sometimes fail in high curvature regions of the point set. This problem was addressed in Moon et al. (2017), and a new method was proposed namely Eq. (15).

2.4. Adjustment of control points

We assume that a point $\mathbf{p} = (x_c, y_c, z_c)$ is given, where $z_c = f(x_c, y_c)$. Then, from Eqs. (1) and (2), we have

$$\phi_{kl} = \alpha z_c, \quad \alpha = \frac{B_k(s)B_l(t)}{\sum_{a=0}^{3}\sum_{b=0}^{3}(B_a(s)B_b(t))^2}. \tag{16}$$

It means that the control point that interpolates z_c is computed by the amount of distance to z_c multiplied by a weight α. This approach can be extended to the adjustment of control points. We assume that $\Delta d_{ij} = |\mathbf{p}_{ij} - \mathbf{p}_{ij}^d|$, where $\mathbf{p}_{ij}^d$ is the projection of $\mathbf{p}_{ij}$ on the point set and $\mathbf{p}_{ij}$ is the point on the surface defined by the control points ϕ_{ij}. Then, the new control point is estimated to be $\phi_{ij}^* = \phi_{ij} + \alpha\Delta d_{ij}$ using (16), which defines an updated surface that closely approximates $\mathbf{p}_{ij}^d$. This adjustment step is applied to all control points one by one, yielding a new surface approximating the point set. The updated surface is then used to compute Δd_{ij}. This process can be repeated until the adjustment is less than the user defined tolerance.

The adjustment of control points proposed in this work reduces the error Δd_{ij} as follows. At the k-th step, we have $\Delta d_{ij}^{(k)} = \mathbf{p}_{ij}^d - \mathbf{p}_{ij}^{(k)}$. Then, we consider two consecutive errors $\Delta d_{ij}^{(k+1)}$ and $\Delta d_{ij}^{(k)}$. Consider the difference of $\Delta d_{ij}^{(k+1)}$ and $\Delta d_{ij}^{(k)}$. Namely,

$$\begin{aligned}\Delta d_{ij}^{(k+1)} - \Delta d_{ij}^{(k)} &= \mathbf{p}_{ij}^d - \mathbf{p}_{ij}^{(k+1)} - (\mathbf{p}_{ij}^d - \mathbf{p}_{ij}^{(k)}),\\ &= \mathbf{p}_{ij}^{(k)} - \mathbf{p}_{ij}^{(k+1)},\\ &= \sum_a\sum_b \phi_{ab}B_a(u_i)B_b(v_j)\\ &\quad - \sum_a\sum_b(\phi_{ab} + \alpha\Delta d_{ij}^{(k)})B_a(u_i)B_b(v_j),\\ &= -\sum_a\sum_b \alpha\Delta d_{ij}^{(k)}B_a(u_i)B_b(v_j).\end{aligned}$$

For $\Delta d_{ij}^{(k)} > 0, \Delta d_{ij}^{(k+1)} - \Delta d_{ij}^{(k)} < 0$ because $\sum_a\sum_b \alpha\Delta d_{ij}^{(k)}B_a(u_i)$ $B_b(v_j) > 0$. Therefore, $\Delta d_{ij}^{(k+1)} < \Delta d_{ij}^{(k)}$, which means that as the adjustment step is repeated, the approximation error decreases. For $\Delta d_{ij}^{(k)} < 0$, a similar conclusion can be drawn. Here, α is the coefficient that controls the convergence speed. In this work, $\alpha = 1.49$ is used. For a 4×4 control points, we have $\alpha \approx 1.49$ with $s = 1/3$ and $t = 1/3$ from (16). This particular value, which shows the best performance, has been chosen empirically through a series of tests with various α's for each $s = 0, 1/3, 2/3, 1$ and $t = 0, 1/3, 2/3, 1$. Using this process, the surface can be adjusted for reducing Δd_{ij}, which can represent more details on the surface and improve the accuracy of the approximation.

Through a series of experiments, it is noticed that the error reduction obtained by the iteration of the adjustment process is not significant in most cases. Therefore, applying the adjustment method once is sufficient in practice. However, if one iteration of adjustment does not satisfy the tolerance, then we run the MBA with a refined control lattice as shown in Fig. 3.

2.5. Measurement of approximation quality

In this study, the quality of approximation of the input point set is measured using the RMSE (Root Mean Square Error). It is based on the Euclidean distance which is the distance between a grid point on the approximation surface and its closest point among the input points. Suppose that we have n_g grid points and the input point set P. Then, the RMSE is defined as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n_g}(Min(||\mathbf{G}_i - \mathbf{P}||))^2}{n_g}}. \quad (17)$$

Eq. (17) is suitable for evaluating the quality of approximation compared to others that may involve curvature. The curvature is a surface-intrinsic property that requires the second derivatives, which are sensitive to the surface smoothness. When a surface is represented by a set of points, distorted curvature values could be obtained because the derivative computation from the discrete points is highly affected by the level of noise in the point set. On the other hand, the Euclidean distance is less sensitive to the noise. Therefore, we used the Euclidean distance based measure in this study.

3. Experimental results and discussion

Three different scattered data sets were used to demonstrate the proposed method. The first example is a scattered data set representing a human face as shown in Fig. 6(a). It consists of 9801 unstructured points. The MBA method with 19 × 19 control points yielded an RMSE value of 0.0905, whereas the proposed method yielded an RMSE value of 0.0902 with the same number of control points. Although the RMSE values by the MBA and proposed methods are similar, the proposed method represented the shape of the data points more accurately. The nose is better approximated by the proposed method, as shown in Fig. 6.

The second example is a data set of a simulated landform. This data set consists of 9801 data points, as shown in Figs. 7 and 8. In this experiment, two different control nets were considered. Fig. 7 shows the approximation results by the MBA and the proposed methods with 13 × 13 control points. The RMSE value of the proposed method (RMSE = 0.0167) is smaller than that of the MBA method (RMSE = 0.0194), and more details are approximated by the proposed method as shown in Fig. 7. In Fig. 8, both methods are compared using 23 × 23 control points. The proposed method yielded a smaller RMSE (0.0099) value than the MBA method (0.0116), and represented more small features.

In the third example, the same landform data set with more points was considered. In this example, the 35 × 35 control points

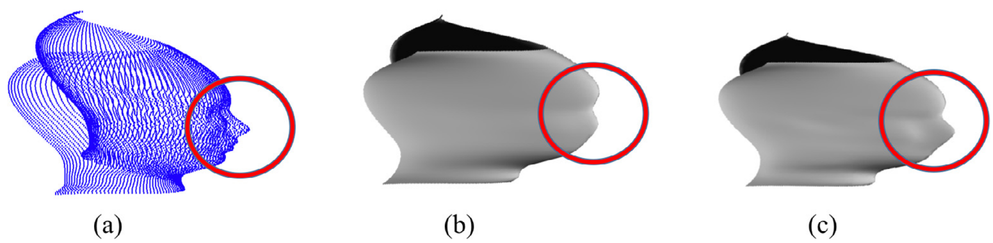


Fig. 6. Result of the multilevel B-spline approximation and the proposed method. (a) The human face scattered data set. (b) Multilevel B-spline approximation (19 × 19 control points, RMSE = 0.0905). (c) The proposed method (19 × 19 control points, RMSE = 0.0904).

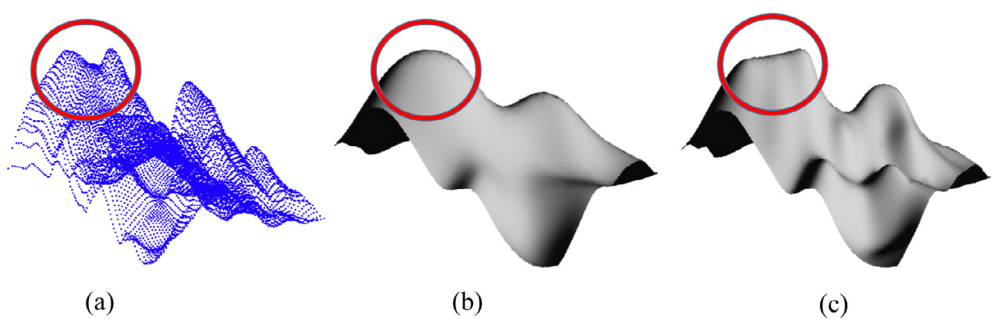


Fig. 7. Result of the multilevel B-spline approximation and the proposed method. (a) Landform scattered data set (N = 9801). (b) Multilevel B-spline approximation (13 × 13 control points, RMSE = 0.0194). (c) The proposed method (13 × 13 control points, RMSE = 0.0178).

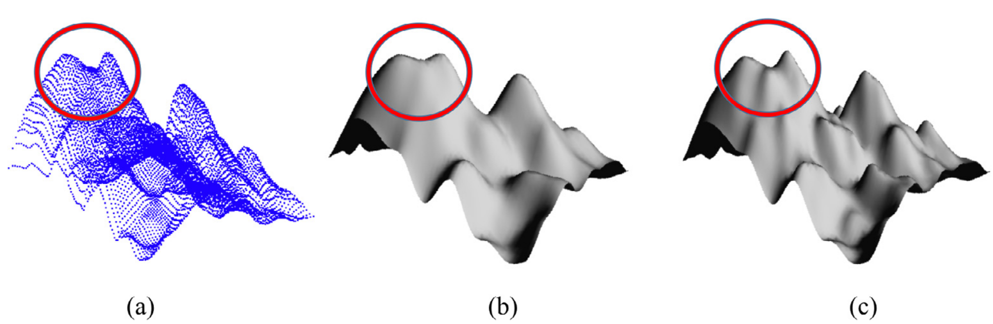


Fig. 8. Result of the multilevel B-spline approximation and the proposed method. (a) Landform scattered data set (N = 249,001). (b) Multilevel B-spline approximation (23 × 23 control points, RMSE = 0.0116). (c) The proposed method (23 × 23 control points, RMSE = 0.0105).

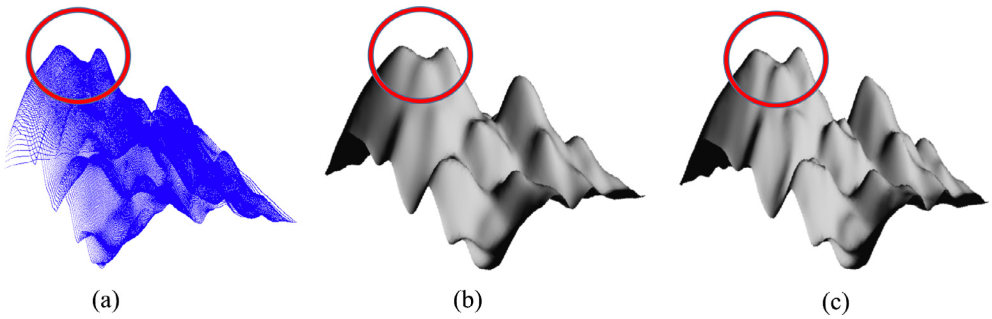
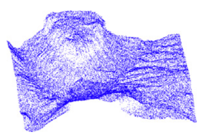
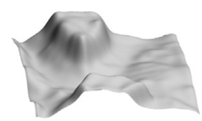
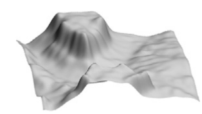
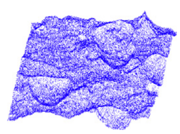
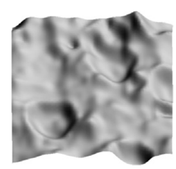
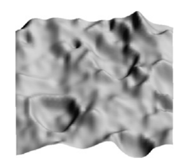
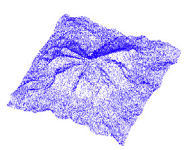
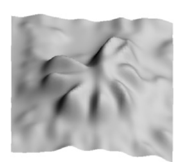
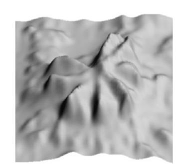
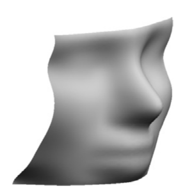


Fig. 9. Result of the multilevel B-spline approximation and the proposed method. (a) Landform scattered data set (N = 249,001). (b) Multilevel B-spline approximation (35 × 35 control points, RMSE = 0.0124). (c) The proposed method (35 × 35 control points, RMSE = 0.0120).

Table 1
More examples. The number of input points, the size of control points, and RMSE for each case are provided.

No.	Input data	MBA	Proposed
1	 50,000 pts	 (35 × 35) RMSE = 0.01741	 (35 × 35) RMSE = 0.01735
2	 45,000 pts	 (35 × 35) RMSE = 0.03521	 (35 × 35) RMSE = 0.03514
3	 40,000 pts	 (35 × 35) RMSE = 0.04284	 (35 × 35) RMSE = 0.03995
4	 11,726 pts	 (19 × 19) RMSE = 0.01050	 (19 × 19) RMSE = 0.00897

were for comparison. The proposed method yielded a smaller RMSE value (0.0119) than the MBA method (0.0124). Moreover, the high curvature regions are better represented by the proposed method, as shown in Fig. 9.

Four more examples were taken to demonstrate the performance of the proposed method compared to the MBA approach as shown in Table 1. As shown in the table, the proposed method produces more accurate results than the MBA method for the same number of control points. In addition, it represents more details of the shape than the MBA method.

4. Conclusion

A novel method for improving the approximation accuracy in the MBA scheme was proposed. The proposed method directly adjusted the control points by considering the errors between the approximated surface generated by the MBA method and the input scattered points.

In contrast to the MBA method, the proposed method increased accuracy without increasing the number of control points. Therefore, the MBA method can avoid further refinement, and the number of control points can be minimized. This can reduce memory requirements and computation time for subsequent processes.

Currently, the weight value for the adjustment of the control points is fixed to be 1.49. However, an optimal weight can be estimated depending on the underlying geometric shape of the points. Moreover, the amount of adjustment can be represented using the multiwavelet method such as Geronimo and Marcellan (2015) without introducing α in the adjustment. The computation of adaptive α and the derivation of an exact representation of the amount of adjustment are recommended for future work.

Conflict of interest

The authors declared that there is no conflict of interest.

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