International Journal of Aeronautical and Space Sciences (2019) 20:1047–1054
https://doi.org/10.1007/s42405-019-00190-5

ORIGINAL PAPER

Fuel Quantity Estimation of Aircraft Supplementary Tank Using Markov Chain Monte Carlo Method

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Received: 28 January 2019 / Revised: 20 May 2019 / Accepted: 21 June 2019 / Published online: 27 June 2019

Abstract

This paper presents an aircraft fuel quantity estimation method using the Markov Chain Monte Carlo (MCMC) method. Using the proposed method, fuel quantity uncertainty of an aircraft supplementary tank can be estimated when the roll and pitch attitudes of an aircraft change. Through reflecting uncertainties, the conservative bound of fuel quantity estimation results can be found, which is necessary for a reliable aircraft operation. The first step of the estimation process is a mathematical modeling of the fuel quantity in a supplementary tank. In the model, the fuel quantity is represented as a multivariate polynomial function of sensor output (i.e., frequency), aircraft roll and pitch angles. The parameter of the mathematical model is then estimated using the MCMC method. As an estimation result, the probability density function of the fuel quantity is provided, which accounts for the uncertainties caused from the developed mathematical model and measured data. The lower bound in the estimation result can be utilized as a conservative fuel quantity value for a reliable operation. To validate the proposed fuel quantity estimation approach, a test with known fuel quantity is performed.

Keywords Fuel Quantity Measurement System (FQMS) · Uncertainty estimation · Bayesian approach · Markov Chain Monte Carlo method

1 Introduction

A supplementary fuel tank for an aerial refueling system enables to increase the range of the aircraft combat and surveillance [1–3]. In the supplementary fuel tank, a fuel quantity measurement system (FQMS) is one of key components. Here, the FQMS functions to estimate the fuel quantity in the situation when an aircraft attitude changes (i.e., roll and pitch angles of an aircraft change). In [4], the FQMS using a capacitance type sensor was developed. The capacitance type sensor measures the height submerged by the fuel by utilizing the difference of the dielectric properties between the air and fuel materials. The FQMS was developed for a unmanned aerial vehicle [5,6]. In [7], the sensor location in the FQMS was optimized to minimize non-measurable fuel quantity.

The previous works for the FQMS [5–7] estimate the fuel quantity based on the deterministic algorithm like a least square regression method. This method finds a single estimation value, and its estimation result does not change for the same input value. The deterministic algorithm has a limitation that it is unable to account for the uncertainty in the estimation process. On the contrary, the probabilistic algorithm can quantify the uncertainty associated with the inaccuracy of an estimation model and measured data. To quantify the uncertainty, the Bayesian approach has been widely utilized due to its practical advantages [8,9]. It should be noted that the uncertainty information in the fuel quantity estimation process is critical for the safe operation of the aircraft.

Accordingly, the Bayesian approach based on Markov chain Monte Carlo (MCMC) method [10–14] is applied for the fuel quantity estimation of aircraft supplementary tank in this work. The MCMC method is one of the effective sampling algorithms that can solve a high-dimensional problem

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using a numerical approximation [10]. It allows us to determine a distribution by randomly sampling values out of the distribution instead of using the mathematical properties of the distribution [14]. The MCMC method has been applied to estimate parameters with uncertainties in various applications. The capacity fade of Li-ion batteries was estimated using the MCMC method in [15,16]. The MCMC method was utilized to estimate the solder alloy material parameters in [17], and reciprocal sliding friction model parameters in [18]. In [19], daily river flow rate was predicted with uncertainty quantification using the MCMC method.

In this work, the MCMC method is applied to estimate the fuel quantity with uncertainties. For the estimation, the fuel quantity is mathematically modeled using the multivariate polynomial function. The model input variables are set as the frequency output of the capacitance type sensor, and roll/pitch angles of the tank. The observation data composed of fuel quantity, sensor output (i.e., frequency), roll and pitch angles are generated using the test simulator. The model parameters are then estimated using the MCMC method with the Metropolis–Hastings algorithm [10–14]. Next, the probability density function (PDF) of the fuel quantity is estimated from the mathematical model and the estimated model parameters. The lower predictive interval bound of the PDF can be utilized as a conservative estimation result of the fuel quantity. To validate the proposed fuel quantity estimation method, the test with known fuel quantity was performed.

The outline of this paper is as follows. In Sect. 2, data generation using the test simulator is explained. In Sect. 3, an explanation about the fuel quantity estimation method is provided. Here, the proposed mathematical model of the fuel quantity is first explained, and the MCMC method utilized for the parameter estimation is briefly described. In Sect. 4, the fuel quantity estimation results are provided to validated the proposed method. Finally, conclusions are provided in Sect. 5.

2 Data Generation Using Test Simulator

To build the numerical model of the fuel quantity, data at various aircraft attitudes are generated using a test simulator. The test simulator is composed of aircraft supplementary tank, attitude motion control equipment and fuel feeding unit, as shown in Fig. 1. Inside the supplementary fuel tank, a passive DC capacitance type sensor is installed to measure the fuel quantity. The output of the sensor is the form of the frequency. Using the test simulator, the sensor data (i.e., frequency) at various fuel quantities, roll and pitch angles are automatically acquired by the control device of the attitude and feeding equipment. The detailed explanations about the test simulator and its fuel quantity measurement system are provided in [20,21].

Table 1 summarizes the data points of fuel quantity Q, roll angle ϕ, and pitch angle θ. The capacity of the supplementary tank is 84L, and thus 28 data points with 3-L intervals are chosen for the fuel quantity Q data points. The ranges of roll and pitch movements are assumed, respectively, as -2 to $2°$ and -3 to $8°$. The data points for the given ranges are determined by the automatic attitude simulation equipment. The equipment is controlled to operate at every $0.5°$ interval inside the given ranges, and thus 9 and 23 data points are, respectively, chosen for the roll and pitch angles. The exact pitch and roll angles at the given data points are measured, as summarized in Table 1. As a result, a total of 5796 data points ($= 28 \times 9 \times 23$ points) are determined, and the sensor output (i.e., frequency f) is measured at each data point.

3 Fuel Quantity Estimation Method

In this section, a fuel quantity estimation method using the Markov chain Monte Carlo (MCMC) method is explained. The first step of the estimation is to build the numerical model to represent the fuel quantity Q as the function of sensor output frequency f, roll angle ϕ, and pitch angle θ. Next, the probability density function (PDF) of the model parameter is estimated using the MCMC method. Then the fuel quantity with uncertainties can be estimated from the numerical model with the estimated model parameters.

3.1 Fuel Quantity Modeling

The numerical model is proposed to represent the fuel quantity Q as a function of sensor frequency f, roll angle ϕ, and pitch angle θ. As explained in Sect. 2, the sensor frequency f is measured as an output during the data generation process because fuel quantity Q, roll angle ϕ, and pitch angle θ are easily controllable quantities. In contrast, the fuel quantity Q is set for the output of the numerical model because it is the final estimated quantity. The first step to build the numerical model is the transformation of the input variables f, ϕ, and θ into $\tilde{f}$, $\tilde{\phi}$, and $\tilde{\theta}$ to have ranges between 0 and 1:

$$\tilde{f} = 2\frac{f - f_{\max}}{f_{\max} - f_{\min}} - 1, \tag{1}$$

$$\tilde{\phi} = 2\frac{\phi - \phi_{\max}}{\phi_{\max} - \phi_{\min}} - 1, \tag{2}$$

$$\tilde{\theta} = 2\frac{\phi - \phi_{\max}}{\phi_{\max} - \phi_{\min}} - 1. \tag{3}$$

After the transformation using (1)–(3), the range of variables $\tilde{f}$, $\tilde{\phi}$, and $\tilde{\theta}$ is restricted between -1 and 1. Then the fuel

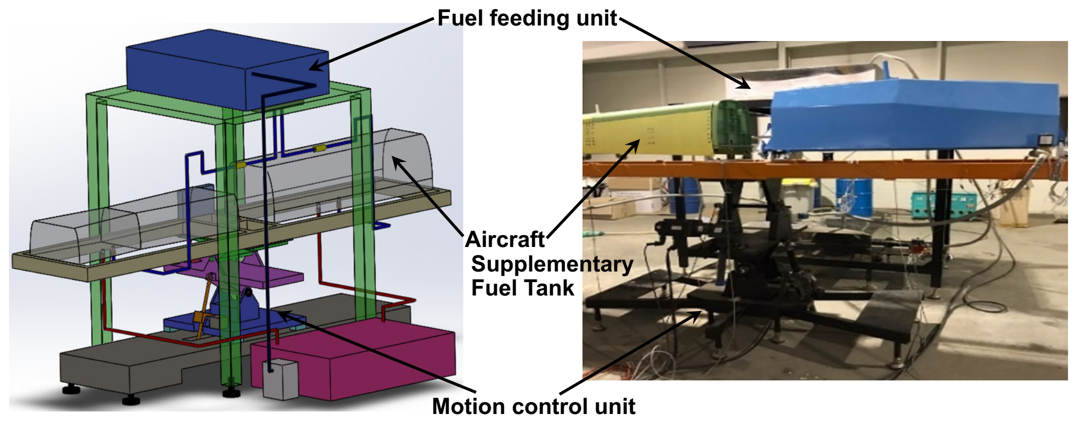

Fig. 1 Test simulator for data generation

Table 1 Summary of data points

Sensor output (frequency f) is measured at data points	
Fuel quantity Q	3L, 6L 9L, 12L, 15L 18L, 21L 24L 27L, 30L, 33L, 36L, 39L, 42L, 45L, 48L, 51L, 54L, 57L, 60L, 63L, 66L, 69L, 72L, 75L, 68L, 81L, 84L
Roll angle ϕ	−1.91°, −1.56°, −1.04°, −0.52°, −0.08°, 0.43°, 0.95°, 1.47°, 1.91°
Pitch angle θ	−2.93°, −2.58°, −2.06°, −1.55°, −1.03°, −0.51°, −0.08°, 0.43°, 0.94°, 1.46°, 1.98°, 2.41°, 2.93°, 3.44°, 3.96°, 4.48°, 4.91°, 5.43°, 5.94°, 6.46°, 6.98°, 7.41°, 7.93°

quantity Q is represented as a summation of polynomial multiplications of transformed input variables $\tilde{f}$, $\tilde{\phi}$, and $\tilde{\theta}$

$$Q(f,\phi,\theta)=\sum_{i=0}^{p}\sum_{j=0}^{p}\sum_{k=0}^{p}C_{ijk}\times\tilde{f}^{i}\times\tilde{\theta}^{j}\times\tilde{\phi}^{k}, \tag{4}$$

where p is the maximum order of polynomials, and C_{ijk} is the model parameter. The larger the maximum order p, the higher the model accuracy, but the higher the calculation cost. For example, when p is one, only eight model parameters from C_{000} to C_{111} are required to complete the model (4). As p increases, the number of the model parameter C_{ijk} increases dramatically. If p is set as 7, the number of model parameter becomes 512 ($= 8 \times 8 \times 8$). To find the smallest p with an acceptable error, the root mean square (RMS) and maximum error between the measured fuel quantity and estimated fuel quantity using (4) are investigated. In this investigation, the model parameter C_{ijk} is determined using the conventional least square method. Figure 2a, b, respectively, describes the calculated RMS, and maximum error with respect to the maximum order of polynomial p. Here the error is defined as the difference between the measured and estimated fuel quantities at 5796 measured data points (refer to Sect. 2). As expected, both RMS and maximum error decreases as the maximum order of polynomials p increases. However, the higher value of p requires the higher computation cost. Thus, the maximum order p is set to 3 in this work considering the accuracy and computational efficiency. It is noted that the number of the model parameter C_{ijk} is 64(=$4 \times 4 \times 4$) when the maximum order of polynomials p in (4) is 3.

To validate the numerical model in (4) with $p = 3$, the measured fuel quantity values are compared with the estimated values acquired using the numerical model (4). The comparison results are provided in Fig. 3a, b. In Fig. 3a, the roll angle ϕ is fixed as −0.08°, and the fuel quantities Q at various frequencies f and pitch angles θ are investigated. On the other hand, Fig. 3b shows the fuel quantities Q at various frequencies f and roll angles ϕ when the pitch angle θ is fixed as 0.43°. This comparison result confirms that the proposed model with $p = 3$ can suitably represent the fuel quantity Q.

Next, the model parameter vector $\mathbf{C}$ composed of the 64(=$4 \times 4 \times 4$) coefficients C_{ijk} in (4) is defined as

$$\mathbf{C}=[C_{000}\quad C_{001}\quad C_{002}\quad \ldots\quad C_{333}]^{\mathrm{T}}. \tag{5}$$

When the model parameter $\mathbf{C}$ is determined, the fuel quantity model (4) is completed. Then the fuel quantity Q can be estimated using (1)–(4) when the sensor frequency f, roll angle ϕ, and pitch angle θ are measured as the input values.

3.2 Markov Chain Monte Carlo Method

The Bayesian inference with the Markov chain Monte Carlo (MCMC) sampling method is utilized to determine the model

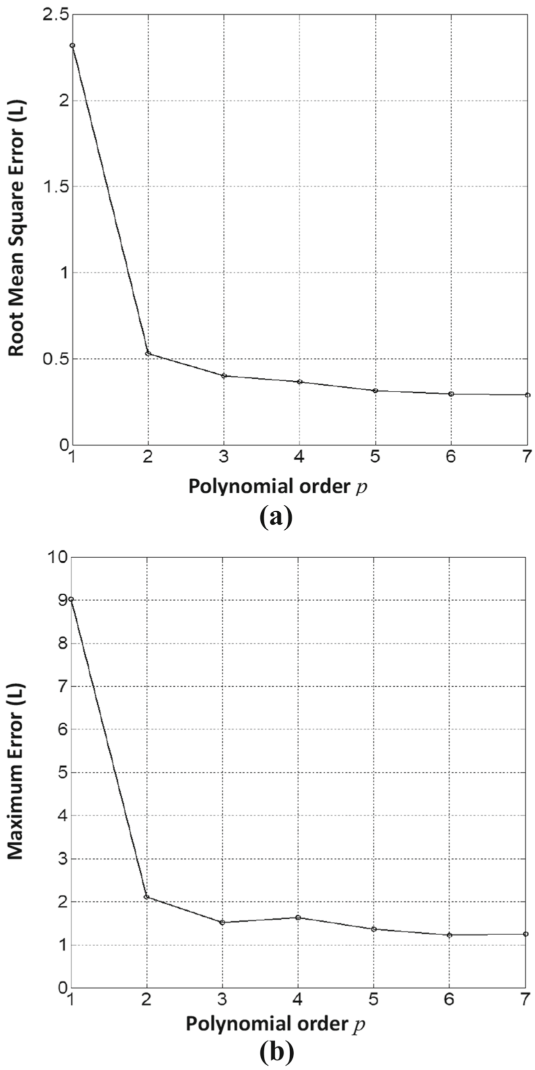

Fig. 2 Calculation of polynomial model error: **a** root mean square error, and **b** maximum error, with respect to polynomial order p

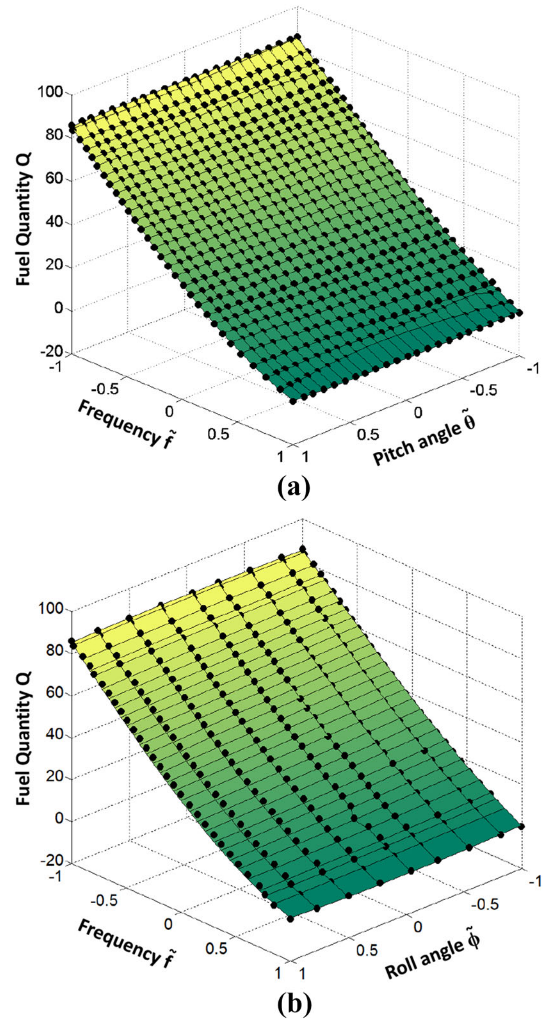

Fig. 3 Comparison of measured and estimated fuel quantities. The black-colored dot represents the measured fuel quantity value, and the surface represents the estimated value using the numerical model (4)

parameter vector **C** with uncertainty. The Bayesian inference uses observed data to update a prior state of beliefs about model parameters to become a posterior state of beliefs about model parameters [14]. In this work, the posterior probability density function (PDF) of the model parameter **C** conditional on the measured fuel quantity data **Q** (i.e., $P(\mathbf{C}|\mathbf{Q})$) can be defined based on the Bayes' rule:

$$P(\mathbf{C}|\mathbf{Q}) \propto L(\mathbf{Q}|\mathbf{C})p(\mathbf{C}), \tag{6}$$

where $p(\mathbf{C})$ is the prior distribution of the model parameter **C**, which is assumed as the uniform distribution in this work. The joint likelihood function $L(\mathbf{Q}|\mathbf{C})$ in (6) is defined as the multiplication of the likelihood functions for 5796 observation data Q_q ($q = 1, 2, ..., 5796$). Here, the 5796 observation data Q_q ($q = 1, 2, ..., 5796$) are acquired using the test simulator, as explained in Sect. 2:

$$\begin{aligned} L(\mathbf{Q}|\mathbf{C}) &= \prod_{q=1}^{5796} L(Q_p|\mathbf{C}) \\ &= L(Q_1|\mathbf{C}) \times L(Q_2|\mathbf{C}) \times \cdots \times L(Q_{5796}|\mathbf{C}). \end{aligned} \tag{7}$$

The likelihood function for qth fuel quantity data (i.e., $L(Q_q|C)$) in (7) is defined as

$$L(Q_q|\mathbf{C}) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{ -\frac{1}{2}\frac{\left[Q_q - Q^q(\mathbf{C}, f_q, \phi_q, \theta_q)\right]^2}{\sigma^2} \right\}, \tag{8}$$

where σ is the standard deviation, and Q^q is the model value of the fuel quantity calculated using (4) with the model parameter **C**, and the observation data f_q, ϕ_q, θ_q. The likelihood function (8) is obtained by assuming that the error of the observation data with respect to the model value is the Gaussian distribution.

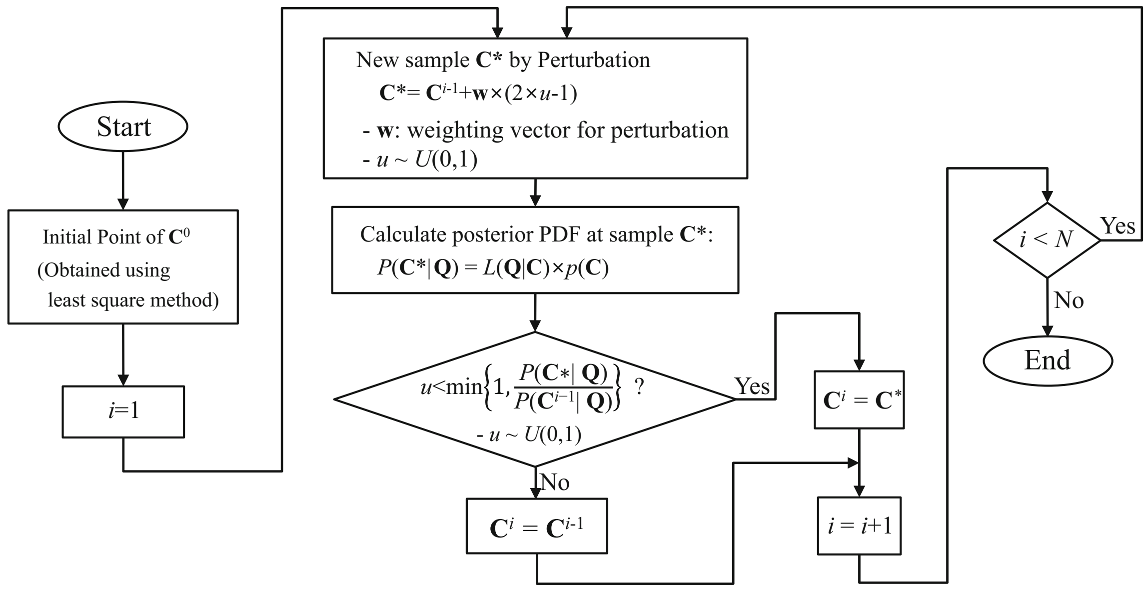

Fig. 4 Flowchart to calculated the PDF of the model parameter **C** using the MCMC method with the Metropolis–Hastings algorithm

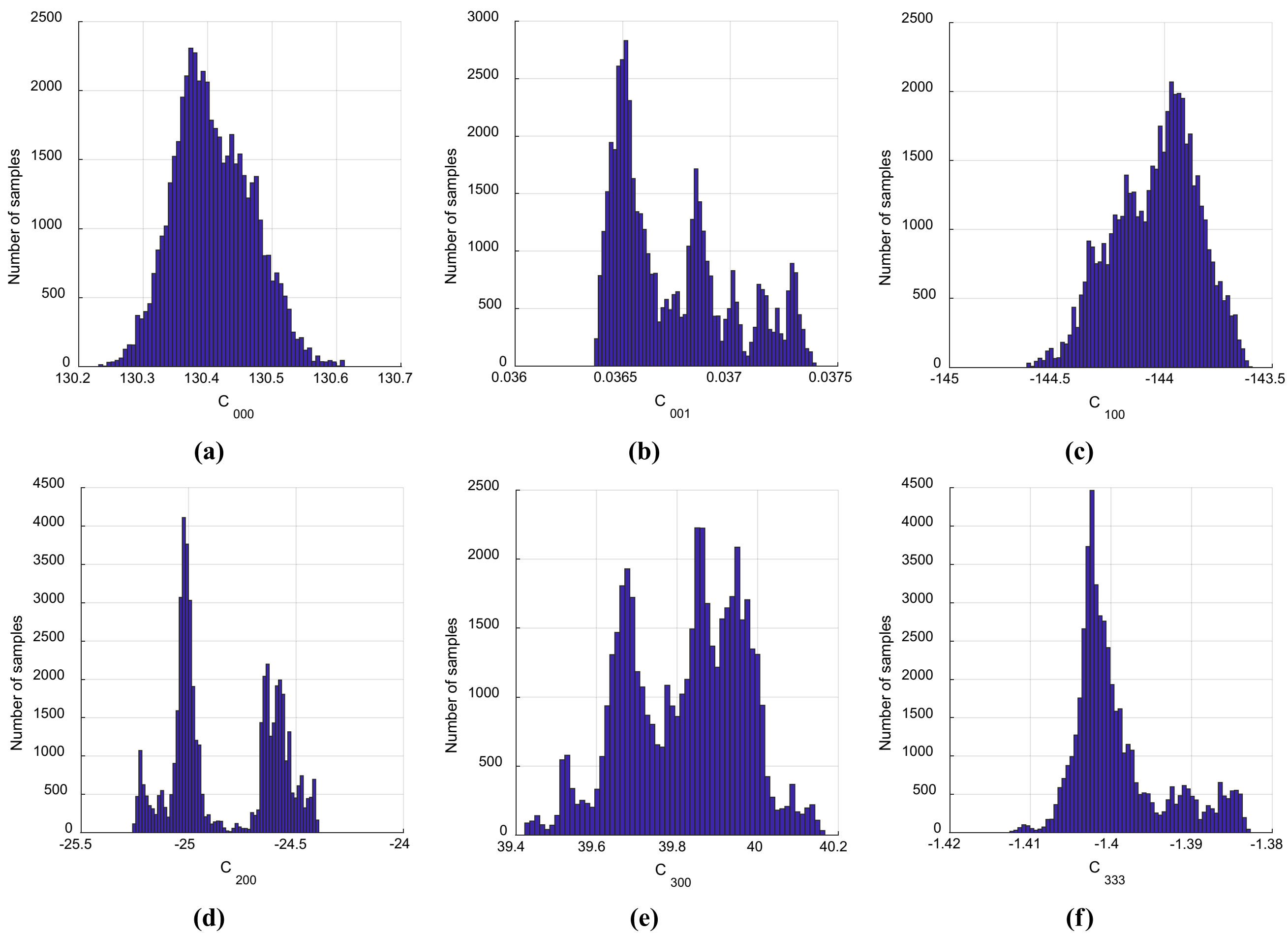

Fig. 5 Estimation result of the model parameter **a** C_{000}, **b** C_{001}, **c** C_{100}, **d** C_{200}, **e** C_{300}, and **f** C_{333}. The histogram shows the number of samples of the Metropolis–Hastings algorithm, which represents the estimated posterior PDF

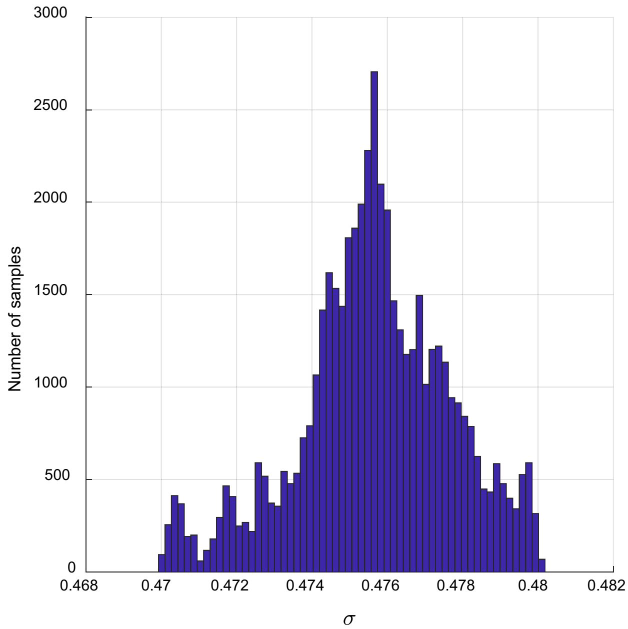

Fig. 6 Estimation result of the standard deviation of the likelihood function in (5). The histogram represents the estimated posterior PDF

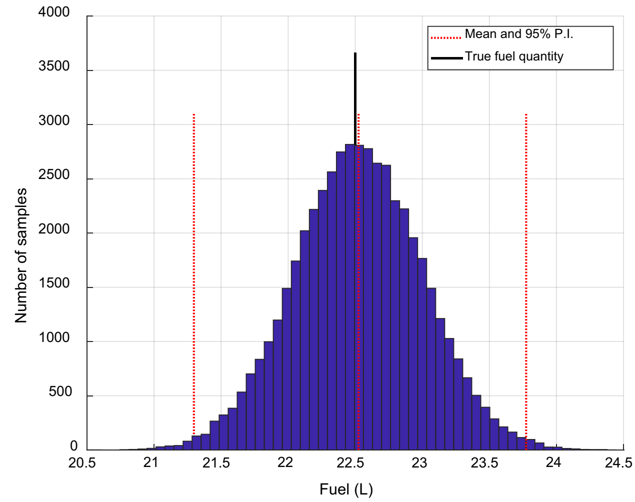

Fig. 7 Estimation result of fuel quantity Q when the true value is 22.5L. The histogram represents the estimated PDF. The mean and upper/lower 95% predictive interval (PI) values are provided as the red-colored dashed lines

To evaluate the posterior PDF of $P(\mathbf{C}|\mathbf{Q})$ in (6) with (7)–(8), the MCMC method using the Metropolis–Hastings algorithm [10–14] is applied in this work. The MCMC method is a computationally efficient sampling approach. It allows us to calculate the posterior PDF $P(\mathbf{C}|\mathbf{Q})$ using the iterative sampling process, instead of analytical derivation. The flowchart of the Metropolis–Hastings algorithm is described in Fig. 4. The first step is to set the initial sample points of the parameter $\mathbf{C}^0$ using the least square method. Next, the new sample $\mathbf{C}^*$ is generated by adding the random noise. The posterior PDF at the new sample $\mathbf{C}^*$ is then calculated. Next, the ratio of posterior PDFs with new sample $\mathbf{C}^*$ and previous sample at $(i-1)$th iteration $\mathbf{C}^{i-1}$ is compared with the random value u to determine whether the sample for ith iteration $\mathbf{C}^i$ is updated with the new sample $\mathbf{C}^*$ or previous sample $\mathbf{C}^{i-1}$. When enough number of samples are acquired, the iteration stops. In this work, the MCMC method with the Metropolis–Hastings algorithm is implemented and run in MATLAB. To understand the detailed principle of the MCMC method with Metropolis–Hastings algorithm, it is recommended to refer to the literature [10–14].

4 Estimation Result

The posterior PDF of the model parameter vector $\mathbf{C}$ conditional on the measured data is estimated using the MCMC method with the Metropolis–Hastings algorithm. Figure 5 shows the estimation results for several of 64 parameters (from C_{000} to C_{333}). In Fig. 5, the number of samples (i.e., y-axis) in the Metropolis–Hastings algorithm represents the estimated posterior PDF. In addition, the estimation result of the standard deviation σ in the likelihood function (8) is provided in Fig. 6. In this work, the total number of samples in the Metropolis–Hastings algorithm is set as 50,000, and the first 1000 samples are discarded as a burn-in period. It should be noted that the estimation result in Fig. 6 incorporates the uncertainties caused from both numerical model and the measurements.

The posterior PDF of the fuel quantity Q is estimated using the numerical model (4) with the estimated model parameter $\mathbf{C}$, when the sensor frequency f, roll angle ϕ, and pitch angle θ are given as the input values. Figure 7 shows the fuel quantity estimation result when the roll angle $\phi = 0.01°$, pitch angle $\theta = -2.672°$, and the frequency $f = 13,276$Hz. The true fuel quantity is set as 22.5L, and the estimated mean value is 22.508L. The upper and lower 95% predictive intervals (i.e., the range in which future observations will be located) are calculated as 21.279L and 23.732L. These values quantify the magnitude of uncertainties in the fuel quantity estimation result. The lower bound value (i.e., 21.279L) can be used as a representative estimation result for a reliable operation. It is noted that the MCMC method provides the PDF (i.e., uncertainty information) as the fuel quantity estimation result, while the conventional deterministic method gives only point-estimated values without the consideration of uncertainty. For example, a single value 22.904L is obtained as the fuel quantity estimation result when applying the trilinear interpolation scheme.

Next, the fuel quantity Q is estimated using the MCMC method when the roll and pitch angles vary with respect to time, as can be seen in Figs. 8a, b and 9a, b. Figures 8c and

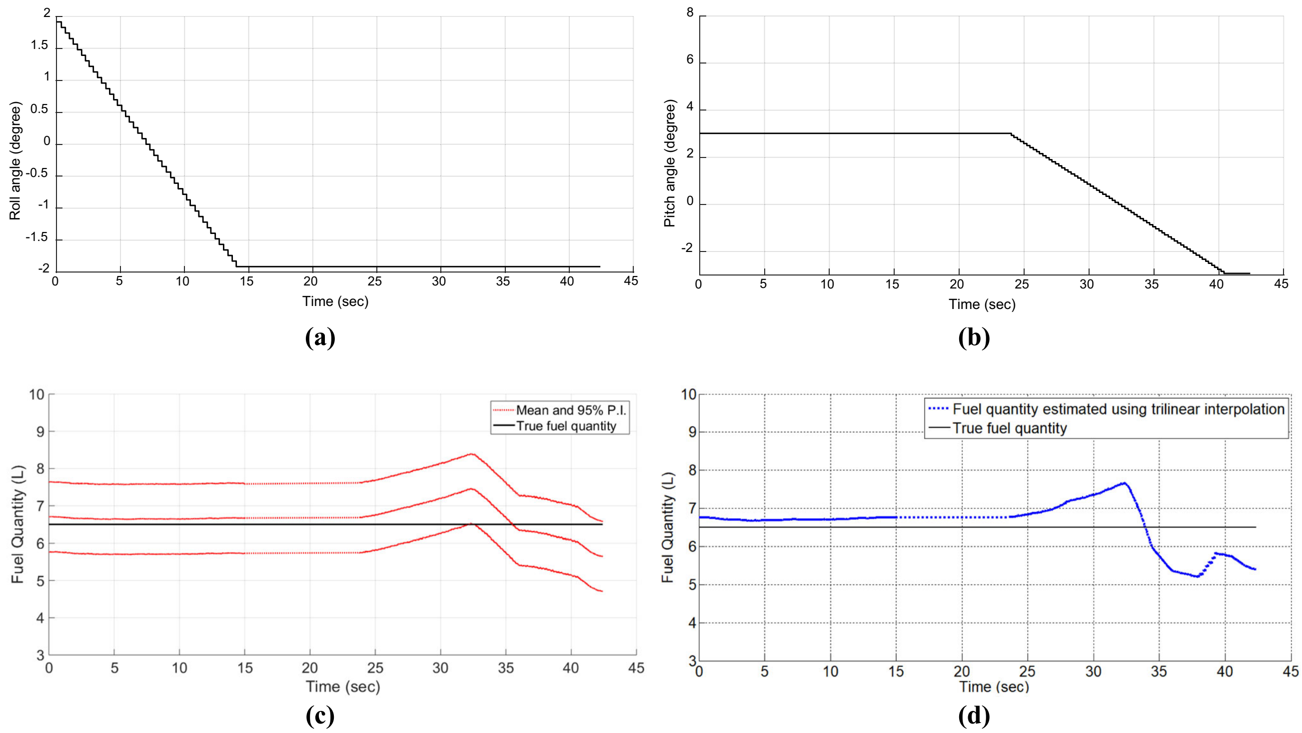

Fig. 8 Fuel quantity estimation result (true fuel quantity = 6.5L). **a** Roll angle ϕ variation with respect to time. **b** Pitch angle θ variation with respect to time. **c** Estimation result obtained using the MCMC method. Mean and 95% upper/lower predictive interval (PI) are provided as red-colored dashed lines. **d** Fuel quantity estimated using the trilinear interpolation scheme

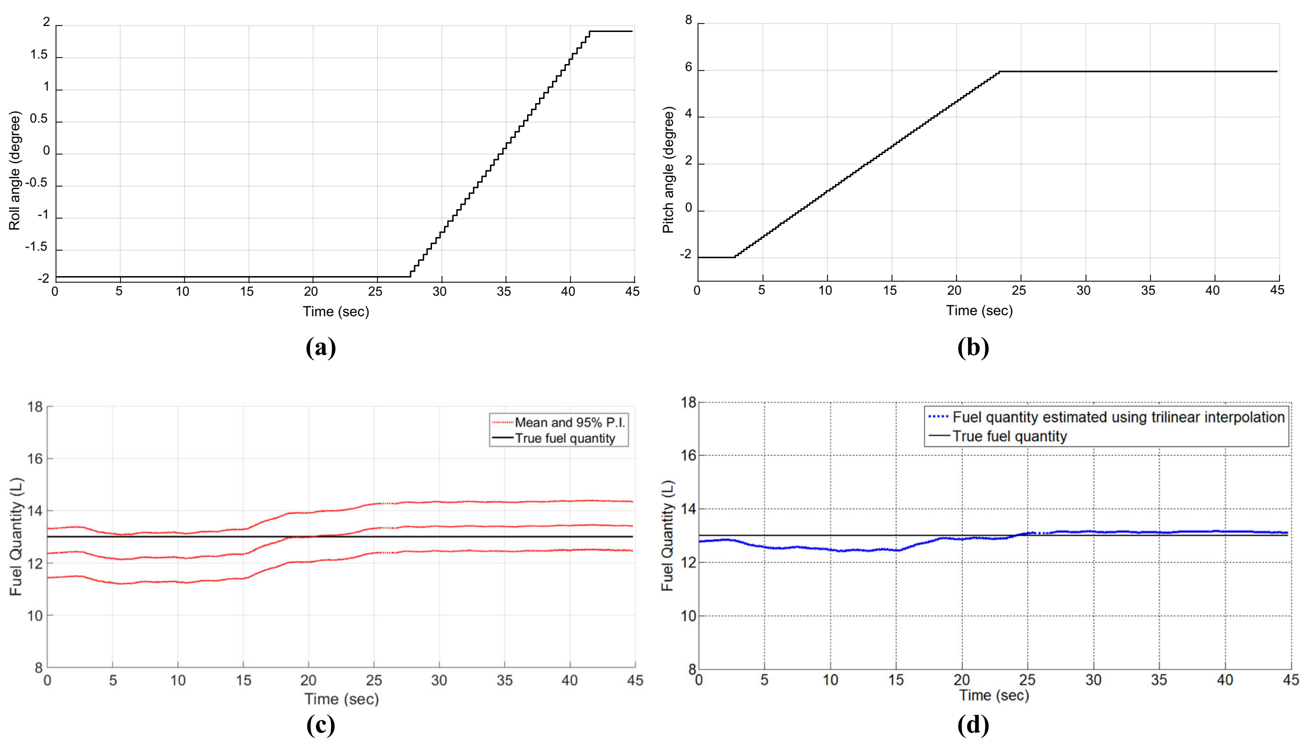

Fig. 9 Fuel quantity estimation result (true fuel quantity = 13L). **a** Roll angle ϕ variation with respect to time. **b** Pitch angle θ variation with respect to time. **c** Estimation result obtained using the MCMC method. Mean and 95% upper/lower predictive interval (PI) are provided as red-colored dashed lines. **d** Fuel quantity estimated using the trilinear interpolation scheme

9c show the estimation result using the proposed MCMC method when the true fuel quantities are 6.5L and 13L, respectively. In both results, the true fuel quantity is located within 95% upper/lower predictive intervals (PI). It is noted again that the 95% predictive intervals mean the range where future observations locate with a 95% probability. Thus, it is confirmed that the numerical model in (4) successfully describes the fuel quantity as the function of input variables (i.e., sensor frequency f, roll angle ϕ, and pitch angle θ), and the fuel quantity can be estimated with the uncertainties using the MCMC method. The lower 95% PI bound could be utilized as the conservative estimation result for better reliability. To clarify the advantage of the proposed approach considering uncertainties, the fuel quantities estimated using the trilinear interpolation scheme are provided in Figs. 8d and 9d. It is noted that the trilinear interpolation scheme is one of deterministic approaches that gives a point-estimated value. It is observed from Fig. 8d that the fuel quantity is considerably overestimated when the time is around 32 s. This overestimation might prevent the safe operation of an aircraft. On the contrary, the overestimation of the fuel quantity does not occur when the lower PI bound of the MCMC estimation result is utilized. The overestimation of the fuel quantity is also observed in Fig. 9d, which confirms the advantage of the proposed approach considering uncertainties.

5 Conclusion

In this work, the uncertainty estimation method of the fuel quantity in a supplementary fuel tank is presented. Unlike the deterministic algorithm (e.g., least square method), the proposed method can quantify the uncertainty caused from the mathematical model and measurement. For the modeling, a multivariate polynomial function is utilized, and the training data are prepared using the test simulation. Then the model parameters are calculated using the MCMC method, and the PDF of the fuel quantity can be obtained as an estimation result. In the test estimation with known fuel quantity, the uncertainties are successfully quantified using the proposed method. The lower bound of the predictive intervals can be utilized as a conservative value considering the uncertainty.

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