



Fixed-time orientation estimation and network localisation of multi-agent systems

Younghun John¹ | Kwang-Kyo Oh² | Bariş Fidan³ | Hyo-Sung Ahn¹

¹ School of Mechanical Engineering, Gwangju Institute of Science and Technology, Gwangju, Republic of Korea

² Department of Electrical Engineering/Smart Energy Institute, Sunchon National University, Sunchon, Republic of Korea

³ Department of Mechanical and Mechatronics Engineering, University of Waterloo, Waterloo, Ontario, Canada

Correspondence

Kwang-Kyo Oh, Department of Electrical Engineering/Smart Energy Institute, Sunchon National University, Sunchon, Republic of Korea. Email: kwangkyo.oh@gmail.com

Abstract

Among different distributed network localisation methods, displacement-based network localisation with orientation estimation is an effective approach because it does not require global information and yet asymptotically produces accurate estimates. In this paper, to improve transient performance of this approach, non-linear laws to achieve fixed-time orientation estimation and fixed-time displacement-based localisation are proposed. A sequential algorithm ensuring fixed-time convergence property and show uniform asymptotic stability of a fixed-time network localisation with simultaneous fixed-time orientation estimation is further provided. Simulation results verify that the proposed laws achieve network localisation of multi-agent systems within fixed settling time.

1 | INTRODUCTION

Locating positions of agents in multi-agent systems has become more and more important in various applications. Particularly, it is indispensable in sensor networks and formation control systems [1]. GPS-based localisation methods have been popularly used because they allow agents to obtain their position information with respect to the global coordinate system. Though the GPS-based methods are effective and easy to use, they have some well-known drawbacks. The drawbacks result from the fact that GPS signals are not always available. GPS signals are not available in indoor and outer space operations. Further terrain conditions such as street canyon and dense vegetation may prevent GPS signals from being utilised [2].

As an alternative method to GPS-based localisation, one can consider estimation of positions of agents based on local measurements. Among a variety of local measurement types, distance, bearing and displacement are widely used in the literature. Distance-based network localisation problems are well formulated in [3, 4], where rigidity of an underlying graph of a given multi-agent system is crucial. Similarly, it has been known that bearing rigidity is essential for bearing-based localisation [5–7]. Though rigidity-based localisation methods are beneficial because they require only distance or bearing measurements, their performance heavily depends on highly dense interconnections of agents, which is linked to the rigidity of underlying graphs. Different from distance- and bearing-based network localisation, displacement-based localisation is based on both inter-agent distance and relative bearing measurements. Though displacement-based method requires more measurements than rigidity-based ones, it is based on uniform connectedness of underlying graphs, which requires less dense interconnections compared to rigidity. Displacement-based method is also beneficial in terms of the number of anchors. In practical network localisation problems, it is desirable to locate positions of agents with respect to the global coordinate system. To achieve the goal, localisation systems need anchors, which can be considered position-aware agents. At least two or three anchors are necessary in distance- and bearing-based systems to ensure localisations of agents with respect to the global coordinate system. However, only one anchor is able to ensure the global localisation in displacement-based approach. In short, displacementbased network localisation is beneficial in some aspects though it requires more measurements compared to rigidity-based ones.

A drawback of displacement-based localisation is that every agent needs to share a common sense of orientation, which

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is not desirable because such information cannot be achieved in distributed manners. The drawback of displacement-based localisation has been mitigated based on the orientation estimation in [8–10]. In those works, agents are allowed to estimate their orientations based on the interaction through sensing and communication topologies. Once the orientation estimation is achieved, localisation can be effectively performed based on displacement-based method. In [8–10], it has been shown that orientation estimation is achieved exponentially if underlying graphs are connected and then localisation is also asymptotically achieved.

In this paper, we mainly focus on displacement-based network localisation with orientation estimation. Particularly, we seek to improve the transient performance of the existing displacement-based localisation algorithm with orientation estimation, thereby ensuring the localisation of agents within fixed settling time. Finite settling-time property was originally encountered in dynamical systems with discontinuous inputs. Despite the benefits, it is difficult to analyse such discontinuous systems and further those systems also have some undesired characteristics. In consequence, researchers have focused on developing finite-time converging controllers that do not cause discontinuities. Those systems have been characterised by finite-time stability [11, 12]. It has been known that finite settling time may depend on initial conditions of dynamical systems. To overcome this drawback, researchers have attempted to design a system that has a settling time with an upper bound independent of initial conditions [13]. The equilibrium point with such property is called fixed-time stable. Recently, fixed-time stability has been studied in multi-agent systems to achieve fast convergence. Finite-time consensus problems have been addressed [14] and developed into controlling the positions of agents [15, 16]. The authors of [7] have showed both orientation estimation and formation control but only designed sequential algorithm. Fixed-time stability for consensus problems have been addressed [17, 18] without further applications.

The fixed-time localisation tasks are aimed to be achieved by proposing non-linear laws for orientation estimation and displacement-based network localisation. It further suggests both sequential and simultaneous operation of the proposed laws. The main contribution of the paper are as follows: (i) a fixed-time stability of the individual orientation estimation and network localisation are verified; (ii) a sequential algorithm of orientation estimation and network localisation is proposed suggesting an upper bound of the settling time determined by the number of agents and few design parameters; (iii) uniform asymptotic stability of an interconnected system of fixed-time orientation estimation and fixed-time network localisation is shown. Simulation results show that fixed-time orientation estimation and network localisation are well achieved using the proposed non-linear laws.

The remaining is organised as follows: In Section 2, we formulate the fixed-time network localisation problem based on displacement measurements and orientation estimation.

Section 3 reviews some existing results and explains our approach to the problem. Then fixed-time orientation estimation algorithm is designed and convergence analysis is provided in Section 4. Fixed-time network localisation with orientation-aware agents is proposed in Section 5. In Section 6, sequentially and simultaneously integrated fixed-time orientation estimation and fixed-time network localisation are considered. Simulation results are provided in Section 7. Concluding remarks follow in Section 8.

2 | PROBLEM STATEMENT

In this section, we state the fixed-time orientation estimation and network localisation problems. Consider *n* agents on the plane. The position and orientation of agent *i*, for i = 1, ..., n, are denoted by

$$p_i = [p_{i\times} p_{iy}]^{\mathsf{T}} \in \mathbb{R}^2,$$
$$\theta_i \in [0, 2\pi),$$

respectively. By stacking the position and orientation variables, we have the following vector variables:

$$P = [p_1^{\mathsf{T}} \ p_2^{\mathsf{T}} \ \cdots \ p_n^{\mathsf{T}}]^{\mathsf{T}} \in \mathbb{R}^{2n},$$
$$\Theta = [\theta_1 \ \theta_2 \ \cdots \ \theta_n] \in [0, \ 2\pi)^n.$$

The corresponding interaction topology between the agents is represented by an undirected graph with nodes representing agents and edges representing both communication and sensing links between agent pairs. It is assumed that the agents do not have access to global measurements of their orientations and positions. Instead, we assume that each agent is able to measure relative angles, δ_{ji} , and relative positions, p_{ji} , of its neighbours with respect to its own reference frame, as illustrated in Figure 1. Also, agents share their own estimates of orientation and positions with neighbouring agents through communication.

Based on the measured angles δ_{ji} , the agents are able to obtain relative bearing, θ_{ii} , which is defined as

$$\theta_{ji} = PV(\theta_j - \theta_i)$$

:= $(\theta_j - \theta_i + \pi) \mod 2\pi - \pi.$ (1)

The following is then obvious.

$$PV(\theta_{j} - \theta_{i}) = PV(\delta_{ji} - \delta_{ij} + \pi).$$
⁽²⁾

Under these conditions, our first goal is to allow the agents to estimate their own orientations with respect to the global reference frame, ${}^g\Sigma$, within a fixed settling time. The final estimates of the orientations might contain a common offset, $\bar{\theta}$, compared to the true values. The detailed definition of $\bar{\theta}$ will





FIGURE 1 Interaction between two neighbour agents

be mentioned in Section 4. Denote $\hat{\theta}_i$ as the estimated orientation value of agent *i*, then the fixed-time orientation estimation problem is formulated as follows:

Problem 1. Consider an *n*-agent system on the plane. Assume that the underlying undirected graph of the system is connected and each agent has access to the orientation estimates and the relative bearings of its neighbouring agents. Design an orientation estimation law to produce, for each agent $i \in \{1, 2, ..., n\}$, the estimate $\hat{\theta}_i$ of the orientation θ_i such that $\hat{\theta}_i \rightarrow \theta_i + \bar{\theta}$ for some constant $\bar{\theta}$ within a fixed settling time.

Since the agents obtain a common sense of orientation within a fixed settling time once Problem 1 is solved, we can assure that the agents are aware of their orientations. Fixed-time network localisation can then be stated for the orientation-aware agents. Define \hat{p}_i as the estimated position of agent *i* with respect to the global reference frame and denote by $R_{\theta} \in \mathbb{R}^{2\times 2}$ the rotation matrix by an angle θ about the origin. Let the superscript represent the corresponding reference frame while we omit the notation for the global reference frame for brevity. Agent *i* can measure the relative positions of its neighbouring agents with respect to its own reference frame, $i \Sigma$,

$$p_{ji}^{i} := p_{j}^{i} - p_{i}^{i}, \tag{3}$$

where i = 1, ..., n and $j \in \mathcal{N}_i$. Then the problem of fixed-time network localisation for orientation-aware agents is stated as follows:

Problem 2. Consider an *n*-agent system on the plane. Assume that the underlying undirected graph is connected and each

agent has access to the position estimates and the relative position of its neighbouring agents. Further assume that each agent is aware of its orientation with a common offset $\bar{\theta}$ to the true value based on the fixed-time orientation estimation. Design a network localisation law that generates the position estimates \hat{p}_i , \hat{p}_j with respect to the global reference frame such that $\hat{p}_j - \hat{p}_i$ converges to $R_{\bar{\theta}}(p_j - p_i)$, $\forall i \in \{1, ..., n\}$, $j \in \mathcal{N}_i$ within a fixed settling time.

Another main purpose of this paper is to integrate a localisation scheme with an orientation estimation law, so that a system of n agents with no global information, estimates the positions of the agents under a common reference frame within a fixed settling time. The problem can be defined as an adaptive localisation protocol where the system considers the orientation system parameter. The problem of designing the integration of orientation estimation and localisation schemes is defined as follows:

Problem 3. Consider an *n*-agent system on the plane. Assume that the underlying undirected graph of the system is connected and each agent has access to the orientation and the position estimates; and the relative bearings and the relative positions of its neighbouring agents. Design an adaptive localisation scheme composed of an orientation estimation law and a network localisation law to generate the position estimates \hat{p}_i , \hat{p}_j with respect to the global reference frame such that $\hat{p}_j - \hat{p}_i$ converges to $R_{\bar{\Phi}}(p_j - p_i)$, $\forall i \in \{1, ..., n\}$, $j \in \mathcal{N}_i$ within a fixed settling time.

3 | THE PROPOSED APPROACH

Consider localisation of a network of multiple agents. It is assumed that the agents are not capable of obtaining their global orientations and global positions. Instead, the agents are able to measure relative distances and relative angular displacements of their neighbouring agents. We study fixed-time network localisation under such conditions. To present the proposed approach, we need to review some preliminary results on distributed consensus algorithm and fixed-time stability.

Consider a system of *n*-agents modelled as single integrators. In distributed state consensus for this system, we are interested in agreement of the state variables of agents. A well-known consensus protocol is as follows [19]:

$$\dot{x}_i = \sum_{j=1}^n a_{ij} (x_j - x_i), \tag{4}$$

where $x_i \in \mathbb{R}$ denotes the state of agent $i \in \{1, 2, ..., n\}$ and a_{ij} is the element of weighted adjacency matrix $A \in \mathbb{R}^{n \times n}$. In the literature, it is known that the protocol (4) achieves consensus of the multi-agent system exponentially fast [19].

While exponential convergence is desirable, it implies that the states of the agents cannot reach consensus within a finite time, which might be undesirable depending on applications. For instance, finite-time orientation estimation and network localisation would be beneficial to formation control if the control strategy is based on the estimated positions. Based on this observation, one can consider how the consensus of the agents could be achieved within finite time. It has been shown that the following non-linear protocol achieves consensus of the agents within finite time [14]:

$$\dot{x}_i = \sum_{j=1}^n a_{ij} (x_j - x_i)^{[\mu]}, \quad 0 < \mu < 1,$$
 (5)

where $x^{[\mu]}$ denotes sign(x) $|x|^{\mu}$.

We review two properties characterising finite-time consensus: finite-time and fixed-time stability. While these properties are conceptually similar, they are different in terms of whether the settling time is bounded regardless of initial conditions or not. Consider a system described by the following non-linear differential equation:

$$\dot{x} = f(t, x), \quad x(0) = x_0,$$
 (6)

where $x \in \mathbb{R}^n$ and $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is possibly discontinuous, which was extensively addressed in [20]. Suppose that the origin is an equilibrium point of the system described in (6). Then the finite-time stability can be defined as follows:

Definition 1 [12], [13]. The origin of (6) is globally finite-time stable equilibrium point of the system if it is globally asymptotically stable and any solution of $x(t, x_0)$ of (6) reaches it within some finite time, that is, $x(t, x_0) = 0 \forall t \ge T(x_0)$, where $T : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ is the settling-time function.

In the definition of the finite-time stability, settling time might vary according to the initial condition or it has an upper bound. If the origin of system (6) is globally finite-time stable but the settling time can vary depending its initial condition, then the origin is called finite-time stable. If the settling time is upper bounded regardless of the initial condition, then the system is further called fixed-time stable. Illustrative behaviours of finiteand fixed-time stable systems are shown in Figure 2. As shown in Figure 2(a), the convergence time of the finite-time stable system is a function of the initial condition. However, fixed-time stable system has an upper bound of the convergence time and the bound is independent of the initial condition as shown in Figure 2(b). Fixed-time stability is mathematically defined as follows:

Definition 2 [13]. The origin of (6) is *fixed-time stable* equilibrium point of the system if it is globally finite-time stable and the settling-time function $T(x_0)$ is bounded, that is, $\exists T_{\text{max}} > 0$: $T(x_0) \leq T_{\text{max}}, \forall x_0 \in \mathbb{R}^n$.

Based on the definitions of finite- and fixed-time stability, we introduce a lemma that will be used in what follows to analyse fixed-time stability of two consensus systems describing orientation estimation and network localisation dynamics:



(b) *Fixed-time stable system*

FIGURE 2 Comparison between (a) finite-time stable system and (b) fixed-time stable system

Lemma 1 [21]. *If there exists a continuous radially unbounded function* $V : \mathbb{R}^n \to \mathbb{R}_+ \cup 0$ *such that*

1. $V(x) = 0 \Leftrightarrow x = 0;$ 2. any solution x(t) of (6) satisfies the inequality $\dot{V}(x(t)) \leq -\alpha V^{\rho}(x(t)) - \beta V^{\sigma}(x(t))$ for some α , $\beta > 0$, $\rho = 1 - \frac{1}{2\gamma}$, $\sigma = 1 + \frac{1}{2\gamma}$, $\gamma > 1$,

then the origin is globally fixed-time stable for system (6) and the following estimate of the settling-time function holds:

$$T(x_0) \leq T_{max} := \frac{\pi \gamma}{\sqrt{\alpha \beta}}, \ \forall x_0 \in \mathbb{R}^n.$$

The concepts of convergence within a fixed time and convergence at a fixed time should be distinguished. In this paper, we are interested in fixed-time control that guarantees convergence within an upper bound of settling time. Fixed-time control that gives an exact settling time has been studied in [22]. Based on input shaping, the authors of [22] have proposed controllers that ensure convergence of a class of multi-agent systems at a specified time.

Given the explanations on consensus algorithm, fixed-time stability and fixed settling time, we propose fixed-time estimation laws in the upcoming sections to solve the problems introduced in Section 2. A fixed-time orientation estimation



FIGURE 3 Illustration on diverse bearing definitions

algorithm is designed in Section 4 to solve Problem 1. A fixed-time network localisation algorithm for orientation-aware agents is designed in Section 5 to solve Problem 2. To solve Problem 3, we integrate two estimation algorithms in two different ways, running them (i) sequentially and (ii) simultaneously and they are introduced in Section 6. Sequentially integrated system utilises the aforementioned two fixed-time estimation laws. In simultaneously integrated system, a fixed-time network localisation scheme is designed for agents with no pre-estimated orientations and is combined with fixed-time orientation estimation law.

4 | FIXED-TIME ORIENTATION ESTIMATION

We first design an orientation estimation law to solve Problem 1 under the assumption that the orientations of agents are not aligned and only the true measurements of the relative bearings of its neighbouring agents, $\hat{\theta}_{ij}$, and the estimated orientations of it neighbouring agents, $\hat{\theta}_j$ are given. Since several new definitions on angles are introduced in Sections 4–6, an illustration describing the variables is provided in Figure 3.

The following consensus-based orientation estimation law can be found in the literature [8-10]:

$$\dot{\hat{\theta}}_i = \sum_{j=1}^n a_{ij} (\hat{\theta}_j - \hat{\theta}_i - \theta_{ji}).$$
⁽⁷⁾

However, the orientation estimation law (7) does not ensure fixed-time convergence, thus the following non-linear estimation law is proposed consisting of two terms with different exponents:

$$\begin{aligned} \dot{\hat{\theta}}_{i} &= \sum_{j=1}^{n} \left[\alpha_{o} \{ a_{ij} (\hat{\theta}_{j} - \hat{\theta}_{i} - \theta_{ji}) \}^{[\mu_{o}]} \right. \\ &+ \beta_{o} \{ a_{ij} (\hat{\theta}_{j} - \hat{\theta}_{i} - \theta_{ji}) \}^{[\nu_{o}]} \right], \end{aligned}$$

$$(8)$$

where $0 < \mu_o < 1$, $\nu_o > 1$, and α_o and β_o are some positive constants.

Define the orientation estimation error variable as

$$\Delta \theta_i(t) := \hat{\theta}_i(t) - \theta_i$$

Then the non-linear estimation law (8) can be written as a nonlinear average consensus protocol introduced in [17]:

$$\Delta \dot{\theta}_{i} := \sum_{j=1}^{n} \left[\alpha_{o} \{ a_{ij} (\Delta \theta_{j} - \Delta \theta_{i}) \}^{[\mu_{o}]} + \beta_{o} \{ a_{ij} (\Delta \theta_{j} - \Delta \theta_{i}) \}^{[\nu_{o}]} \right], \tag{9}$$

In order to utilise Lemma 1, denote the average value of the estimation error values as

$$\bar{\theta} = \frac{1}{n} \sum_{i=1}^{n} \Delta \theta_i(t_0), \qquad (10)$$

and define a disagreement vector $\tilde{\Theta} = [\tilde{\theta}_1 \cdots \tilde{\theta}_n]^{\mathsf{T}}$ as

$$\tilde{\theta}_i(t) := \Delta \theta_i(t) - \bar{\theta}.$$
(11)

Then (9) can be rewritten based on the new variables as

$$\dot{\tilde{\theta}}_{i} = \sum_{j=1}^{n} \left[\alpha_{o} \{ a_{ij} (\tilde{\theta}_{j} - \tilde{\theta}_{i}) \}^{[\mu_{o}]} + \beta_{o} \{ a_{ij} (\tilde{\theta}_{j} - \tilde{\theta}_{i}) \}^{[\nu_{o}]} \right].$$
(12)

Define ψ_{ij} as follows:

$$\psi_{ij}(\tilde{\theta}_j - \tilde{\theta}_i) = \alpha_{\delta} \{a_{ij}(\tilde{\theta}_j - \tilde{\theta}_i)\}^{[\mu_{\delta}]} + \beta_{\delta} \{a_{ij}(\tilde{\theta}_j - \tilde{\theta}_i)\}^{[\nu_{\delta}]},$$

which leads to

$$\hat{\tilde{\theta}}_i = \sum_{j=1}^n \psi_{ij} (\tilde{\theta}_j - \tilde{\theta}_i).$$

Note that ψ_{ij} can be considered an action function introduced in [23], which satisfies the following property:

$$\psi_{ij}(\Delta\theta_j - \Delta\theta_i) = -\psi_{ji}(\Delta\theta_i - \Delta\theta_j).$$
(13)

Due to (11), we have $\Delta \dot{\theta}_i(t) = \dot{\theta}_i(t)$. From (13), we further obtain the following:

$$\sum_{i=1}^{n} \Delta \dot{\theta}_i = 0, \qquad (14)$$

which implies

$$\frac{1}{n}\sum_{i=1}^{n}\Delta\theta_{i}(t) = \frac{1}{n}\sum_{i=1}^{n}\Delta\theta_{i}(t_{0}) = \bar{\theta}$$
(15)

and

$$\sum_{i=n}^{n} \tilde{\theta}_i(t) = 0, \ \forall t > 0.$$
(16)

This shows that the origin is the equilibrium point of (12). In short, $\Delta\Theta$ converges to $\bar{\theta}\mathbf{1}_n$ and by the definition of the estimation error variable, the orientation estimation law (8) converges to the true orientation value with a common offset $\bar{\theta}$. It is worth emphasising that although $\bar{\theta}$ has been introduced in the process of deriving the equilibrium point of (12), the knowledge on the value $\bar{\theta}$ is not needed to be known by the agents for fixed-time convergence of the proposed estimation scheme. Then we have the following result:

Theorem 1. Consider a system of n agents whose orientations are not aligned. Assume that the underlying graph of the system is undirected and connected. Under the orientation estimation protocol (8) with $\mu_o = 1 - \frac{1}{\gamma_o}, \nu_o = 1 + \frac{1}{\gamma_o}, \gamma_o > 1$, the error dynamics (9) converges to its equilibrium set within the maximum settling time,

$$T_{max}^{o} = \frac{\pi \gamma_{o} \pi^{\frac{1}{2\gamma_{o}}}}{2\sqrt{\alpha_{o}\beta_{o}} (\lambda_{*}(\mathcal{L}_{\mu_{o}}))^{\frac{1}{2} - \frac{1}{4\gamma_{o}}} (\lambda_{*}(\mathcal{L}_{\nu_{o}}))^{\frac{1}{2} + \frac{1}{4\gamma_{o}}}}$$

where λ_* is the second smallest eigenvalue of a Laplacian matrix and \mathcal{L}_{μ_o} and \mathcal{L}_{ν_o} represent the Laplacian matrices of weighted graphs with adjacency matrices $A^{\left[\frac{2\mu_o}{\mu_o+1}\right]}$ and $A^{\left[\frac{2\nu_o}{\nu_o+1}\right]}$, respectively.

Proof. The proof follows Theorem 5 in [17], thus here an outline is given. Define the Lyapunov function as

$$V(\tilde{\Theta}) = \frac{1}{2}\tilde{\Theta}^{\mathsf{T}}\tilde{\Theta} = \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_{i}^{2}.$$
(17)

Then the time derivative of the Lyapunov function is given as

$$\dot{V}(\tilde{\Theta}) \leq -2^{\mu_0} \alpha_o(\lambda_*(\mathcal{L}_{\mu_0}))^{\frac{\mu_0+1}{2}} V^{\frac{\mu_0+1}{2}} -2^{\nu_0} \beta_o n^{1-\nu_0} (\lambda_*(\mathcal{L}_{\nu_0}))^{\frac{\nu_0+1}{2}} V^{\frac{\nu_0+1}{2}}, \quad (18)$$

which can be arranged as

$$\dot{V}(\tilde{\Theta}) \leq -\bar{\alpha}_{o} V^{\rho_{o}}(\tilde{\Theta}) - \bar{\beta}_{o} V^{\sigma_{o}}(\tilde{\Theta}), \tag{19}$$

where

$$\begin{split} \bar{\alpha}_o &= 2^{\mu_o} \alpha_o(\lambda_*(\mathcal{L}_{\mu_o}))^{\frac{\mu_o+1}{2}}, \\ \bar{\beta}_o &= 2^{\nu_o} \beta_o n^{1-\nu_o} (\lambda_*(\mathcal{L}_{\nu_o}))^{\frac{\nu_o+1}{2}}, \\ \rho_o &= \frac{\mu_o+1}{2}, \ \sigma_o &= \frac{\nu_o+1}{2}. \end{split}$$

Based on Lemma 1, it is concluded that the error dynamics (12) is globally fixed-time stable and the upper bound of the settling time is given as

$$T_{\max}^{o} = \frac{\pi \gamma_{o} n^{\frac{1}{2\gamma_{o}}}}{2\sqrt{\alpha_{o}\beta_{o}}\lambda_{*}(\mathcal{L}_{\mu_{o}})^{\frac{1}{2}-\frac{1}{4\gamma_{o}}}\lambda_{*}(\mathcal{L}_{\nu_{o}})^{\frac{1}{2}+\frac{1}{4\gamma_{o}}}},$$
(20)

which completes the proof.

5 | FIXED-TIME NETWORK LOCALISATION OF ORIENTATION-AWARE AGENTS

Assume that the agents are aware of their orientations with respect to the global reference frame with a common offset based on the fixed-time orientation estimation proposed in the previous section. This means that after each agent measures the relative positions of its neighbouring agents with respect to its own reference frame, they correspond to the relative positions based on a common reference frame. The network localisation problem described in Problem 2 will therefore become a displacement-based observation problem [24].

Denote by \hat{p}_{ik} the estimated position of agent *i* along *k*-axis and $R_{k,\theta}$ the *k*th row of the rotation matrix by θ where $k \in \{x, y\}$, that is, $\hat{p}_i = [\hat{p}_{ix}, \hat{p}_{iy}]^{\mathsf{T}}$, $R_{\theta} = [R_{x,\theta}^{\mathsf{T}}, R_{y,\theta}^{\mathsf{T}}]^{\mathsf{T}}$. Also define $\hat{\theta}_i^* := \theta_i + \bar{\theta}$ as the final estimated orientation of agent *i*. Then using the final orientation estimate $\hat{\theta}_i^*$ and the relative position p_{ji}^i , along with the estimated position of its neighbouring agents, \hat{p}_j , the proposed fixed-time network localisation protocol is designed as follows:

$$\dot{\hat{p}}_{ik} = \sum_{j=1}^{n} \left[\alpha \{ a_{ij} (\hat{p}_{jk} - \hat{p}_{ik} - R_{k,\hat{\theta}_{i}^{*}} p_{ji}^{i}) \}^{[\mu]} + \beta \{ a_{ij} (\hat{p}_{jk} - \hat{p}_{ik} - R_{k,\hat{\theta}_{i}^{*}} p_{ji}^{i}) \}^{[\nu]} \right],$$

$$= \sum_{j=1}^{n} \left[\alpha \{ a_{ij} (\hat{p}_{jk} - \hat{p}_{ik} - R_{k,\bar{\theta}} p_{ji}) \}^{[\mu]} + \beta \{ a_{ij} (\hat{p}_{jk} - \hat{p}_{ik} - R_{k,\bar{\theta}} p_{ji}) \}^{[\nu]} \right], \qquad (21)$$

where $\alpha, \beta \in \mathbb{R}_+$, $0 < \mu < 1$, and $\nu > 1$.

Define the error variable as $\Delta p_i := \hat{p}_i - R_{\bar{\theta}} p_i$. Then we have the following error dynamics:

$$\begin{aligned} \Delta \dot{p}_{ik} &:= \sum_{j=1}^{n} [\alpha (a_{ij} (\Delta p_{jk} - \Delta p_{ik}))^{[\mu]} \\ &+ \beta (a_{ij} (\Delta p_{jk} - \Delta p_{ik}))^{[\nu]}]. \end{aligned}$$
(22)

$$\bar{p}_k = \frac{1}{n} \sum_{i=1}^n \Delta p_{ik}(t_0), \quad \bar{p} = \begin{bmatrix} \bar{p}_x \\ \bar{p}_y \end{bmatrix}$$
(23)

and define a disagreement vector \tilde{p}_{ik} as

$$\tilde{p}_{ik}(t) := \Delta p_{ik}(t) - \bar{p}_k, \quad \tilde{p}_i = \begin{bmatrix} \tilde{p}_{ix} \\ \tilde{p}_{iy} \end{bmatrix}.$$
(24)

Then (22) becomes

$$\tilde{p}_{ik} = \sum_{j=1}^{n} [\alpha (a_{ij} (\tilde{p}_{jk} - \tilde{p}_{ik}))^{[\mu]} + \beta (a_{ij} (\tilde{p}_{jk} - \tilde{p}_{ik}))^{[\nu]}].$$
(25)

Define ϕ_{ijk} as follows:

$$\phi_{ijk}(\tilde{p}_{jk} - \tilde{p}_{ik}) = \alpha (a_{ij}(\tilde{p}_{jk} - \tilde{p}_{ik}))^{[\mu]} + \beta (a_{ij}(\tilde{p}_{jk} - \tilde{p}_{ik}))^{[\nu]},$$

which leads to

$$\dot{\tilde{p}}_{ik} = \sum_{j=1}^{n} \phi_{ijk} (\tilde{p}_{jk} - \tilde{p}_{ik}).$$

Note that ϕ_{ijk} can be considered an action function satisfying $\phi_{ijk}(\cdot) = -\phi_{jik}(\cdot)$. From (24), we have $\dot{p}_{ik}(t) = \Delta \dot{p}_{ik}(t)$. Due to the property of the action function ϕ_{ijk} , we further have $\sum_{i=1}^{n} \Delta \dot{p}_{ik} = 0$. This implies

$$\frac{1}{n}\sum_{i=1}^{n}\Delta p_{ik}(t) = \frac{1}{n}\sum_{i=1}^{n}\Delta p_{ik}(t_0) = \bar{p}_k$$
(26)

and

$$\sum_{i=1}^{n} \tilde{p}_{ik}(t) = 0, \quad \forall t > 0.$$
(27)

Thus, the origin is the equilibrium point of (25) and moreover $\Delta p_{ik}(t) \rightarrow \bar{p}_k$ as $t \rightarrow \infty$. So the final estimates of the position of agent *i*, \hat{p}_i , becomes $\bar{p} + R_{\bar{\theta}}p_i$. The following theorem summarises the fixed-time stability of the network localisation error dynamics (25):

Theorem 2. Consider the network localisation protocol (21). Assume that the underlying graph of the agents is undirected and connected. If the parameters μ and ν satisfy $\mu = 1 - \frac{1}{\gamma}$, $\nu = 1 + \frac{1}{\gamma}$, for some $\gamma > 1$, then the error dynamics (22) converges to its equilibrium set within the maximum settling time,

$$T_{max}^{p} = \frac{\pi \gamma n^{\frac{1}{2\gamma}}}{2^{1-\frac{1}{4\gamma}} \sqrt{\alpha \beta} (\lambda_{*}(\mathcal{L}_{\mu}))^{\frac{1}{2}-\frac{1}{4\gamma}} (\lambda_{*}(\mathcal{L}_{\nu}))^{\frac{1}{2}+\frac{1}{4\gamma}}}$$

Proof. Similar steps to the proof of Theorem 1 can be taken to show that the network localisation dynamics is globally fixed-time stable. Denote $[\tilde{p}_1^T \quad \tilde{p}_2^T \quad \cdots \quad \tilde{p}_n^T]$ by \tilde{P} . Take the following Lyapunov function to derive the fixed value of the maximum settling time:

$$V(\tilde{P}) = \frac{1}{2} \tilde{P}^{\top} \tilde{P} = \frac{1}{2} \sum_{k \in \{x, y\}} \sum_{i=1}^{n} \tilde{p}_{ik}^{2}.$$
 (28)

The derivative of the Lyapunov function becomes

$$\dot{V}(\tilde{P}) = -\frac{1}{2} \alpha \sum_{k \in \{x,y\}} \sum_{i,j=1}^{n} a_{ij}^{\mu} |\tilde{p}_{jk} - \tilde{p}_{ik}|^{\mu+1} - \frac{1}{2} \beta \sum_{k \in \{x,y\}} \sum_{i,j=1}^{n} a_{ij}^{\nu} |\tilde{p}_{jk} - \tilde{p}_{ik}|^{\nu+1}.$$

Utilising the norm equivalence property [21]

$$\|\chi\|_{l} \le \|\chi\|_{r} \le n^{\frac{1}{r} - \frac{1}{l}} \|\chi\|_{l}$$
 for $\chi \in \mathbb{R}^{n}$, $l > r > 0$,

which is derived from Hölder's inequality, along with the given conditions $\mu + 1 < 2$, $\nu + 1 > 2$, we obtain

$$\dot{V}(\tilde{P}) \leq -\frac{1}{2} \alpha \left(\sum_{k \in \{x,y\}} \sum_{i,j=1}^{n} a_{ij}^{\frac{2\mu}{\mu+1}} (\tilde{p}_{jk} - \tilde{p}_{ik})^2 \right)^{\frac{\mu+1}{2}} - 2^{-\frac{\nu+1}{2}} \beta n^{1-\nu} \left(\sum_{k \in \{x,y\}} \sum_{i,j=1}^{n} a_{ij}^{\frac{2\nu}{\nu+1}} (\tilde{p}_{jk} - \tilde{p}_{ik})^2 \right)^{\frac{\nu+1}{2}}.$$

If $\tilde{P} \neq \mathbf{1}_{2n}$, we have the following inequalities:

$$\sum_{k \in \{x,y\}} \sum_{i,j=1}^{n} a_{ij}^{\frac{2\mu}{\mu+1}} (\tilde{p}_{jk} - \tilde{p}_{ik})^2 = 2\tilde{P}^{\mathsf{T}} (\mathcal{L}_{\mu} \otimes I_2) \tilde{P}$$

$$\geq 4\lambda_* (\mathcal{L}_{\mu}) V (\tilde{P}),$$

$$\sum_{k \in \{x,y\}} \sum_{i,j=1}^{n} a_{ij}^{\frac{2\nu}{\nu+1}} (\tilde{p}_{jk} - \tilde{p}_{ik})^2 = 2\tilde{P}^{\mathsf{T}} (\mathcal{L}_{\nu} \otimes I_2) \tilde{P}$$

$$\geq 4\lambda_* (\mathcal{L}_{\nu}) V (\tilde{P}),$$

where \mathcal{L}_{μ} and \mathcal{L}_{ν} denote Laplacian matrices corresponding to the weighted adjacency matrices with entries $a_{ij}^{\frac{2\mu}{\mu+1}}$ and $a_{ij}^{\frac{2\nu}{\nu+1}}$, respectively. Then the time derivative of the Lyapunov function becomes

$$\dot{V}(\tilde{P}) \leq -2^{\mu} \alpha (\lambda_*(\mathcal{L}_{\mu}))^{\frac{\mu+1}{2}} V^{\frac{\mu+1}{2}} - 2^{\frac{\nu+1}{2}} \beta n^{1-\nu} (\lambda_*(\mathcal{L}_{\nu}))^{\frac{\nu+1}{2}} V^{\frac{\nu+1}{2}}.$$

By arranging the equation, we further obtain

$$\dot{V}(\tilde{P}) \leq -\bar{\alpha} V^{\rho}(\tilde{P}) - \bar{\beta} V^{\sigma}(\tilde{P}),$$

where

$$\begin{split} \bar{\alpha} &= 2^{\mu} \alpha (\lambda_*(\mathcal{L}_{\mu}))^{\frac{\mu+1}{2}}, \\ \bar{\beta} &= 2^{\frac{\nu+1}{2}} \beta n^{1-\nu} (\lambda_*(\mathcal{L}_{\nu}))^{\frac{\nu+1}{2}}, \\ \rho &= \frac{\mu+1}{2}, \quad \sigma = \frac{\nu+1}{2}. \end{split}$$

This shows that the Lyapunov function satisfies the condition in Lemma 1. Further the upper bound of settling time is given as

$$T_{\max}^{p} = \frac{\pi \gamma n^{\frac{1}{2\gamma}}}{2^{1 - \frac{1}{4\gamma}} \sqrt{\alpha \beta} (\lambda_{*}(\mathcal{L}_{\mu}))^{\frac{1}{2} - \frac{1}{4\gamma}} (\lambda_{*}(\mathcal{L}_{\nu}))^{\frac{1}{2} + \frac{1}{4\gamma}}}, \qquad (29)$$

which completes the proof.

6 | INTEGRATION OF ORIENTATION ESTIMATION AND LOCALISATION SCHEMES

In the previous sections, fixed-time estimation laws for orientation and position estimation were developed separately. To achieve the ultimate goal of designing an adaptive localisation scheme consisting of an orientation estimation law and a network localisation law that solve Problem 3, we introduce two different ways to integrate the orientation estimation and network localisation laws developed in the previous sections; (i) sequentially and (ii) simultaneously.

6.1 | Sequential integration

One way of integrating the fixed-time orientation estimation and network localisation laws is to operate them sequentially as shown in Algorithm 1. The algorithm is designed to perform the fixed-time network localisation after calculating the maximum settling time of the fixed-time orientation estimation. However, computing the maximum settling time of the system requires the information of the system graph topology and edge weights. Since we consider a distributed estimation and localisation scheme, we use the upper bound for the maximum settling time, which can be calculated using the information of the number of agents only. The settling time in (20) is

ALGORITHM 1 Sequential fixed-time orientation estimation and network localisation

Input: <i>n</i> - number of agents in the system	$\alpha_0, \beta_0, \gamma_0$ - coefficients on fixed-time orientation estimation
1:	Compute the minimum possible algebraic connectivity of the connected <i>n</i> -agent system
2:	For the computed λ_* , compute the maximum possible settling time of the fixed-time orientation estimation
3:	for $t_0 \leq t \leq T^o_{Max}$, do
4:	Fixed-time orientation estimation
5:	end for
6:	Fixed-time network localisation



FIGURE 4 An example of an interaction topology between five agent system

inversely proportional to the algebraic connectivity of the underlying graph, thus we aim to obtain the minimum value of the algebraic connectivity. From the previous results studied in [25, 26], we know that the algebraic connectivity is minimum when the corresponding graph is a path graph. Also under the same graph topology, the algebraic connectivity decreases as the weights of the edges decrease. Based on these facts, if we assume that the non-zero weights are greater than a constant, w_c , it is possible to obtain the upper bound of the maximum settling time with only the information of the number of agents.

6.2 | Simultaneous integration

Another way to integrate the two fixed-time estimation laws is to conduct the process simultaneously. As different from the sequential integration, the stability of the simultaneously



 $\label{eq:FIGURE 5} FIGURE \ 5 \qquad \mbox{Orientation estimation error in conventional orientation estimation of the system in Figure 4}$



FIGURE 6 Orientation estimation error in fixed-time orientation estimation of the system in Figure 4

conducted estimation system must be shown because the disagreement value in orientation estimation, $\tilde{\theta}_i$ acts as a disturbance in fixed-time network localisation and may cause finite-time escape in the overall system. In this subsection, we prove uniform asymptotic stability of the simultaneously conducted system and derive an upper bound of the settling time. Local input-to-state stability and uniform asymptotic stability of interconnected systems are well described in the following lemmas:

Lemma 2 [27]. Consider the system $\dot{x} = f(x, u)$. Suppose that in some neighbourhood of (x = 0, u = 0), the function f(x, u) is continuously differentiable. If the unforced system f(x, 0) has a uniformly asymptotically stable equilibrium point at the origin x = 0, then the system f(x, u) is locally input-to-state stable.



FIGURE 7 Position estimation error in conventional network localisation of the system in Figure 4 with orientation-aware agents



FIGURE 8 Position estimation error in fixed-time network localisation of the system in Figure 4 with orientation-aware agents

Consider the interconnected system

$$\dot{x}_1 = f_1(x_1, x_2) \tag{30}$$

$$\dot{x}_2 = f_2(x_2).$$
 (31)

Lemma 3 [27]. If the system (30), with x_2 as input, is locally input-tostate stable and the origin of (31) is uniformly asymptotically stable, then the origin of the interconnected system (30, 31) is uniformly asymptotically stable.

In simultaneously integrated system, the orientations are not estimated beforehand, thus the fixed-time network localisation law is different from the estimation scheme introduced in Section 5. The fixed-time network localisation law with



FIGURE 9 Sequential fixed-time orientation estimation and network localisation of the system in Figure 4



FIGURE 10 Position estimation errors in sequential fixed-time orientation estimation and network localisation of the system in Figure 4

simultaneous fixed-time orientation estimation follows

$$\dot{\hat{p}}_{ik} = \sum_{j=1}^{n} \left[\alpha \{ a_{ij} (\hat{p}_{jk} - \hat{p}_{ik} - R_{k,\Delta\theta_i} p_{ji}) \}^{[\mu]} + \beta \{ a_{ij} (\hat{p}_{jk} - \hat{p}_{ik} - R_{k,\Delta\theta_i} p_{ji}) \}^{[\nu]} \right].$$
(32)

The error dynamics can be written as

$$\begin{split} \Delta \dot{p}_{ik} &= \sum_{j=1}^{n} \left[\alpha \{ a_{ij} (\Delta p_{jk} - \Delta p_{ik} + R_{k,\bar{\theta}} p_{ji} - R_{k,\Delta\theta_i} p_{ji}) \}^{[\mu]} \\ &+ \beta \{ a_{ij} (\Delta p_{jk} - \Delta p_{ik} + R_{k,\bar{\theta}} p_{ji} - R_{k,\Delta\theta_i} p_{ji}) \}^{[\nu]} \right], \end{split}$$
(33)



FIGURE 11 Simultaneous fixed-time orientation estimation and network localisation of the system in Figure 4



FIGURE 12 Position estimation errors in simultaneous fixed-time orientation estimation and network localisation of the system in Figure 4

and further the disagreement dynamics is expressed as

$$\begin{split} \dot{\tilde{p}}_{ik} &= \sum_{j=1}^{n} \left[\alpha \{ a_{ij} (\tilde{p}_{jk} - \tilde{p}_{ik} + (R_{k,\bar{\theta}} - R_{k,\Delta\theta_i}) p_{ji}) \}^{[\mu]} \\ &+ \beta \{ a_{ij} (\tilde{p}_{jk} - \tilde{p}_{ik} + (R_{k,\bar{\theta}} - R_{k,\Delta\theta_i}) p_{ji}) \}^{[\nu]} \right] \\ &= \sum_{j=1}^{n} \left[\alpha \{ a_{ij} (\tilde{p}_{jk} - \tilde{p}_{ik} + (R_{k,\bar{\theta}} - R_{k,\bar{\theta}+\bar{\theta}_i}) p_{ji}) \}^{[\mu]} \\ &+ \beta \{ a_{ij} (\tilde{p}_{jk} - \tilde{p}_{ik} + (R_{k,\bar{\theta}} - R_{k,\bar{\theta}+\bar{\theta}_i}) p_{ji}) \}^{[\nu]} \right] (34) \end{split}$$

Then (12) and (34) forms an interconnected system where $\tilde{\Theta}$ is considered input in (34). The following theorem describes the





FIGURE 13 Simultaneously conducted fixed-time orientation estimation and network localisation of the system in Figure 4 with one anchor node

uniform asymptotic stability of the origin of the interconnected system (12) and (34):

Theorem 3. Assume the origin of the disagreement dynamics (12) and (25) is fixed-time stable, respectively. Then the origin of the interconnected system described by (12) and (34) is uniformly asymptotically stable.

Proof. From Definitions 1 and 2, the fixed-time stability assures global asymptotic stability. Note that (25) is the unforced system of (34). According to Lemma 2, system (34) is locally input-to-state stable. Based on Lemma 3, it is concluded that the origin of the interconnected system described by (12) and (34) is uniformly asymptotically stable.

By the orientation estimation law (8), the orientation estimates converge to their final values within the fixed settling time T_{max}^o . Additionally, the unforced system (25) has the upper bound of the settling time T_{max}^p , which is independent of initial condition. This ensures that the simultaneously integrated system has the upper bound of the settling time $T_{\text{max}} = T_{\text{max}}^o + T_{\text{max}}^p$.

Remark 1. Although the fixed-time estimation laws in this paper are designed for undirected and connected communication graphs, similar laws can also be designed for directed strongly connected communication graphs. Refer to [18] for the proof of fixed-time average consensus over directed communication graphs. The key idea in establishing fixed-time consensus for undirected graphs is the symmetry property of the Laplacian matrix. In the case of a directed graph whose Laplacian matrix is asymmetric, [28] introduced a virtual graph called the mirror graph, having symmetric Laplacian matrix when the communication graph to be directed spanning tree would further cover general cases but become a challenging problem.

Remark 2. Problem 1 and 2 focus on the fixed-time stability of orientation estimation and network localisation. In the process of estimation, there might exist external disturbances in measurements of relative orientations and positions and the communication between agents. In practical applications, such disturbances are inevitable and research upon the subject is desirable to ensure robustness of the system. In the literature [29, 30], it has been shown that PI or PID protocols ensure robustness of general consensus system. Especially, the integral term in the protocol removes the steady-state error resulted from the disturbances. Future research to adopt the idea into fixed-time state estimation systems will ensure correct estimation in the presence of disturbances.

7 | SIMULATION RESULTS

We provide simulation results to validate the proposed fixedtime orientation estimation and network localisation protocols. For simulation, we consider five agents on the plane. We assume that the agents are located in the form of a pentagon shape. The underlying graph capturing the interaction between the agents based on sensing and communication is illustrated in Figure 4 and the weight of each edge is set to 1.

Further we assume that the orientations of the agents are randomly assigned. For the proposed fixed-time orientation and network localisation protocols, parameters are designed as follows: α_0 , β_0 , α , $\beta = 1$, γ_0 , $\gamma = 2$.

We first compare the proposed fixed-time orientation estimation law with the conventional law (7). The weight of each connected edge in the conventional system is also set to 1. Figures 5 and 6 show the orientation estimation error variables under the existing law and the proposed one, respectively. The line indicating the maximum settling time of the fixed-time orientation estimation is also drawn in Figure 6, showing that the orientation estimation error converges to a common offset within the fixed time under the proposed law. $T_{\text{max}}^o \approx 3.3993$ is calculated using (20). Both the estimation error dynamics in (9) and the one found in [8] achieve average consensus with identical offset.

Next we compare the proposed fixed-time network localisation law with the existing one described by

$$\dot{\hat{p}}_{i} = \sum_{i=1}^{n} a_{ij} (\hat{p}_{j} - \hat{p}_{i} - R_{\bar{\theta}} p_{ji}).$$
(35)

The asymptotic convergence property of the system having the agents with the corresponding estimation law is found in [1]. Assuming that each agent in both systems is aware of its orientations, Figures 7 and 8 show the position estimation error variables under the existing law and the proposed one, respectively. $T_{\text{max}}^{p} \approx 3.7070$ obtained from (29) is indicated in Figure 8. This shows that the convergence speed of the proposed law is faster compared to that of the conventional one.

Third, Figures 9 and 10 show the result when the fixed-time orientation estimation and network localisation are sequentially performed. The final estimated positions in Figure 9 show the

pentagon shape with a common offset of $[\bar{p}_x, \bar{p}_y]$ and a rotation by the angle of $\bar{\theta}$ as expected. Figure 10 shows that the fixedtime position estimation is carried out after the upper-bound of the maximum settling time of the fixed-time orientation estimation as calculated in Algorithm 1. We assumed that the edges with non-zero weight had the value $0.5 \leq w_{ij} \leq 1$.

Next, Figures 11 and 12 show the result when the fixed-time orientation estimation and network localisation are simultaneously performed. Figure 11 shows that the orientations and positions are estimated. Figure 12 shows that position estimation error converges to a common offset expectedly. Figure 12 shows that the position estimation error starts to converge before the orientation estimates are fully converged to their final values. This shows that the simultaneously operated system requires a shorter settling time compared to the sequentially operated system.

Lastly, we provide an example that shows how the proposed system can be applied to practical situations by allowing an agent to be aware of global information. Figure 13 shows the simultaneously conducted fixed-time orientation estimation and network localisation under the assumption that agent 1 is aware of its global orientation and position. Expectedly, each position estimate converges to its true value.

8 | CONCLUSIONS

We have studied fixed-time orientation estimation and fixedtime network localisation of planar multi-agent systems. We have developed estimation laws that ensures orientation estimation and network localisation within fixed time, respectively, if the underlying graph of the multi-agent system is undirected and connected. We have designed an algorithm based on the sequential operation of the developed orientation estimation and network localisation laws, which ensure fixed-time convergence of the angle and position estimation error dynamics to their common equilibrium bias values, for example, convergence of the local coordinate frame estimation errors to zero. The uniform asymptotic stability of the interconnected system of fixed-time orientation estimation and fixed-time network localisation has been proved. Based on simulation results, we have showed that the simultaneous operation of the proposed laws result in better performance.

Although we have verified that the proposed system converges faster and within a fixed time, there still remains open problems. It is necessary to study not only convergence speed but also system performance such as disturbance rejection and transient characteristics. Thus it will be important to conduct experimental analysis as well as theoretical study.

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ORCID

Kwang-Kyo Oh https://orcid.org/0000-0003-1020-7286

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