

# Finite-Time Bearing-Based Maneuver of Acyclic Leader-Follower Formations

Minh Hoang Trinh<sup>1</sup>, *Member, IEEE*, and Hyo-Sung Ahn<sup>2</sup>, *Senior Member, IEEE*

**Abstract**—This letter proposes two finite-time bearing-based control laws for acyclic leader-follower formations. The leaders in formation move with a bounded continuous reference velocity and each follower controls its position with regard to three agents in the formation. The first control law uses only bearing vectors, and finite-time convergence is achieved by properly selecting two state-dependent control gains. The second control law requires both bearing vectors and communications between agents. Each agent simultaneously localizes and follows a virtual target. Finite-time convergence of the desired formation under both control laws is proved by mathematical induction and supported by numerical simulations.

**Index Terms**—Bearing-only measurements, formation control, directed acyclic graph.

## I. INTRODUCTION

FOR MANY years, much research efforts have been put on understanding the mechanisms of collective behaviors displayed in nature and realizing them in large-scale systems such as robotic-, sensor-, traffic-, and electrical networks [1]. A notable application is formation control, where a team of autonomous agents (UAVs, UUVs, mobile robots, etc) is required to achieve and maintain a desired formation shape. Different solutions have been proposed to the problem based on various assumptions on the sensing/controlling/communication variables and topologies among the agents [2].

In the bearing-based approach, the main sensing and controlling variables among the agents are the bearing vectors (aka the directional information) or the subtended angles which can be obtained from low-cost cameras [3]–[5]. Bearing-only formation control of a stationary formation with undirected [6], [7] and special directed topologies such as

acyclic leader-follower [8], [9] or directed cycle [10] have been studied in the literature. In flocking control, the agents simultaneously form a desired formation and agree on their velocities. To achieve flocking behavior, the agents sense the relative geometric variables and the relative velocity with regard to a few followers [11]–[13]. Formation tracking is more demanding since it requires the agents to achieve a target formation and follow a few leaders, whose velocities can be time-varying. The ability to maintain a moving target formation shape is crucial for engineering applications such as search-and-rescue, truck platooning, or flight maneuvering. The authors in [14] proposed a control law for single and double-integrator agents using the relative positions and relative velocities. Bearing-only formation tracking with constant leaders' velocity has been studied for single integrators [15], [16], double integrators [15], [17], or robotics agents [15], [18]–[20]. However, bearing-only formation tracking with time-varying leader's velocity has not been studied in the literature.

This work considers the bearing-only maneuver problem for directed acyclic leader-follower formations where the leaders' velocity is a bounded continuous function, thus filling a gap in the literature. The directed acyclic leader-follower structure can somehow describe the V-shape formation in immigrating birds, where each bird only sees and follows several individuals (immediate leaders) in its sight [21]. Thus, the interacting graph has a hierarchical structure and the formation is led by a small number of leaders [22], [23]. To ease the analysis, it is assumed that during the formation maneuver, the positions of the immediate leaders are not collinear. Each follower, modeled by a single-integrator, controls its position based on information obtained from exactly three agents. First, a finite time bearing-only formation maneuver control law is proposed. Finite-time convergence of the target formation is achieved by appropriately choosing two control gains based on the measured bearing vectors. Note that this method has not been introduced in existing formation tracking laws in the literature [24]–[26]. Second, in case the agents can communicate with each others, an estimation-based control strategy is proposed. Each agent determines a virtual target point based on the desired bearing vectors and the received position estimates of the immediate leaders. Simultaneously, the agent tracks its target point and sends this information to its followers. Although this control strategy requires more information than just the sensed bearing vectors, the followers have some freedom to choose their own trajectories to reach the target point. Thus, collision avoidance between agents or obstacle

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Minh Hoang Trinh is with the Department of Automatic Control, School of Electrical Engineering, Hanoi University of Science and Technology, Hanoi 11615, Vietnam (e-mail: minh.trinhhoang@hust.edu.vn).

Hyo-Sung Ahn is with the School of Mechanical Engineering, Gwangju Institute of Science and Technology, Gwangju 61005, South Korea (e-mail: hyosung@gist.ac.kr).

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can be included under this approach. Finally, simulations are given to support the analysis.

The remainder of this letter is organized as follows. Section II contains background and formulates the problem studied in this letter. Section III proposes and studies the bearing-only formation tracking law. The target-point based formation tracking strategy is investigated in Section IV. Section V contains simulation results and Section VI concludes this letter.

## II. PRELIMINARIES AND PROBLEM FORMULATION

### A. Preliminaries

**1) Notations:** The set of real numbers is denoted by  $\mathbb{R}$ . Given  $x \in \mathbb{R}$ , the signum function  $\text{sign}(x)$  takes value 1, 0, -1 if  $x > 0$ ,  $x = 0$ , and  $x < 0$ , respectively. For  $\alpha > 0$ ,  $\text{sig}(x)^\alpha = \text{sign}(x)|x|^\alpha$ , where  $|x|$  is the absolute value of  $x$ . Also, let  $\mathbf{x} \in \mathbb{R}^d$ , one defines  $|\mathbf{x}|^\alpha = \sum_{i=1}^d |x_i|^\alpha$ . The kernel and image of a matrix  $\mathbf{A} \in \mathbb{R}^{n \times n}$  are denoted by  $\ker(\mathbf{A})$  and  $\text{im}(\mathbf{A})$ . The vec operator is defined as  $\text{vec}(\mathbf{a}_1, \dots, \mathbf{a}_n) = [\mathbf{a}_1^\top, \dots, \mathbf{a}_n^\top]^\top$ . Let  $\mathbf{A} \in \mathbb{R}^{d \times d}$  be a symmetric matrix, one uses  $\lambda_i(\mathbf{A})$  to denote the  $i$ -th smallest eigenvalue of  $\mathbf{A}$ .

Let  $\mathbf{x}$  be a nonzero vector in  $\mathbb{R}^d$ , defining the projection matrix  $\mathbf{P}_x = \mathbf{I}_d - \frac{\mathbf{x}\mathbf{x}^\top}{\|\mathbf{x}\|^2}$ . For any vector  $\mathbf{y} \in \mathbb{R}^d$ ,  $\mathbf{P}_x\mathbf{y}$  is the projection of  $\mathbf{y}$  onto the orthogonal space of  $\text{im}(\mathbf{x})$ .  $\mathbf{P}_x$  is symmetric, positive semidefinite,  $\ker(\mathbf{P}_x) = \text{im}(\mathbf{x})$ , and in addition to a zero eigenvalue,  $\mathbf{P}_x$  has  $n - 1$  eigenvalues 1.

**2) Graph Theory:** A directed graph  $\mathcal{G}$  consists of a vertex set  $\mathcal{V} = \{1, \dots, n\}$  of  $|\mathcal{V}| = n$  vertices and an edge set  $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$  of  $|\mathcal{E}| = m$  edges. The neighbor set of a vertex  $i \in \mathcal{V}$  is defined as  $\mathcal{N}_i = \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}\}$ . A simple path is a sequence of edges in  $\mathcal{E}$  connecting vertices in  $\mathcal{V}$  so that there is no repeated vertex (excepting for possibly the first and the last vertices) and edges. If the first and the last vertices of a path coincide, it is called a directed cycle.  $\mathcal{G}$  is a directed acyclic graph if it does not contain any directed cycle. Indexing the edges so that  $\mathcal{E} = \{e_1, \dots, e_m\}$ , the incidence matrix  $\mathbf{H} = [h_{ki}] \in \mathbb{R}^{m \times n}$  has  $h_{ki} = -1$  if  $i$  is the starting vertex of  $e_k$ ,  $h_{ki} = 1$  if  $i$  is the end vertex of  $e_k$ , and  $h_{ki} = 0$ , otherwise.

### B. Problem Formulation

Consider a formation of  $n$  agents in the  $d$ -dimensional space ( $d \geq 2$ ). Each agent in the formation has a local coordinate system  $^i\Sigma$ , and the axes of these local coordinate systems are aligned to each other. The position of agent  $i$ , written in a global coordinate system  $^s\Sigma$ , is denoted by  $\mathbf{p}_i \in \mathbb{R}^d$ .

A formation is characterized by  $(\mathcal{G}, \mathbf{p})$ , where  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  represents both the sensing and control interactions among agents and  $\mathbf{p} = \text{vec}(\mathbf{p}_1, \dots, \mathbf{p}_n) \in \mathbb{R}^{dn}$  is a configuration of the formation. If  $\mathbf{p}_i \neq \mathbf{p}_j$ , the bearing vector between  $i$  and  $j$  is defined as  $\mathbf{g}_{ij} = \frac{\mathbf{p}_j - \mathbf{p}_i}{\|\mathbf{p}_j - \mathbf{p}_i\|} = \frac{\mathbf{z}_{ij}}{\|\mathbf{z}_{ij}\|}$ , where  $\mathbf{z}_{ij}$  is the displacement between agents  $i$  and  $j$ . If  $(i, j) \in \mathcal{E}$ , agent  $i$  can sense  $\mathbf{g}_{ij}$  and control a desired bearing vector  $\mathbf{g}_{ij}^*$  with regard to agent  $j$ . The set of desired bearing vectors in the formation is denoted by  $\Gamma = \{\mathbf{g}_{ij}^*\}_{(i,j) \in \mathcal{E}}$ . It is assumed that  $\Gamma$  is feasible, i.e., there exists a target configuration  $\mathbf{p}^* = \text{vec}(\mathbf{p}_1^*, \dots, \mathbf{p}_n^*) \in \mathbb{R}^{dn}$  satisfying all the bearing vectors in  $\Gamma$ .

Following the definition in [6],  $(\mathcal{G}, \mathbf{p})$  and  $(\mathcal{G}, \mathbf{q})$  are bearing equivalent if and only if  $\mathbf{P}_{(\mathbf{p}_i - \mathbf{p}_j)}(\mathbf{q}_i - \mathbf{q}_j) = \mathbf{0}$ ,  $\forall (i, j) \in \mathcal{E}$ . They

are bearing congruent if and only if  $\mathbf{P}_{(\mathbf{p}_i - \mathbf{p}_j)}(\mathbf{q}_i - \mathbf{q}_j) = \mathbf{0}$ ,  $\forall i, j \in \mathcal{V}$ ,  $i \neq j$ . A formation  $(\mathcal{G}, \mathbf{p})$  is globally bearing rigid if any formation  $(\mathcal{G}, \mathbf{q})$  bearing equivalent to  $(\mathcal{G}, \mathbf{p})$  is also bearing congruent to it.

The augmented bearing rigidity matrix is defined as  $\tilde{\mathbf{R}}_B = \text{diag}(\mathbf{P}_{\mathbf{g}_k})(\mathbf{H} \otimes \mathbf{I}_d)$ . The formation  $(\mathcal{G}, \mathbf{p})$  is infinitesimally bearing rigid if and only if the kernel of  $\tilde{\mathbf{R}}_B$  is only spanned by infinitesimal bearing rigid motions, i.e.,

$$\ker(\tilde{\mathbf{R}}_B) = \text{im}(\mathbf{1}_n \otimes \mathbf{I}_d, \mathbf{p}) = \text{im}(\mathbf{1}_n \otimes \mathbf{I}_d, \mathbf{p} - \mathbf{1}_n \otimes \bar{\mathbf{p}}),$$

where  $\bar{\mathbf{p}} = (\mathbf{1}_n^\top \otimes \mathbf{I}_d)\mathbf{p}/n$  is the formation centroid.

Suppose that in the formation, there are  $l \geq 3$  leaders moving under the following equation

$$\dot{\mathbf{p}}_i = \mathbf{v}_i, \quad i \in \mathcal{V}_l = \{1, \dots, l\}, \quad (1)$$

where  $\mathbf{v}_i$  is the velocity of the leader  $i$ . Denote  $\mathbf{p}^l = \text{vec}(\mathbf{p}_1, \dots, \mathbf{p}_l)$ , and  $\mathbf{v}^l = \text{vec}(\mathbf{v}_1, \dots, \mathbf{v}_l)$ . The remaining agents are followers, which are modeled by single-integrators

$$\dot{\mathbf{p}}_i = \mathbf{u}_i, \quad \forall i \in \mathcal{V}_f = \{l+1, \dots, n\}, \quad (2)$$

where  $\mathbf{p}_i, \mathbf{u}_i \in \mathbb{R}^d$  are respectively the position and the control input of agent  $i$ .

The following assumptions are adopted in this letter.

**Assumption 1:** The leaders are not in collinear positions and they move with the same bounded continuous reference velocity  $\mathbf{v}^*(t)$  satisfying  $\|\mathbf{v}^*(t)\| < \beta$ ,  $\forall t \geq 0$ . Furthermore, no collision happens between the agents.

**Assumption 2:** The followers cannot sense the leaders' velocity but have information on the upper bound  $\beta$ . The bearing sensing and controlling graph  $\mathcal{G}$  is a directed graph generated by the following procedure:

- Starting with  $l$  vertices  $1, \dots, l$  where  $l \geq 3$ .
- For each  $l+1 \leq i \leq n$ , inserting vertex  $i$  together with  $r \geq 3$  new edges  $(i, j_k)$ , where  $j_k \in \{1, \dots, i-1\}$ .

Obviously, any graph  $\mathcal{G}$  satisfies Assumption 2 is directed acyclic (see Fig. 1(a) for an example).

**Assumption 3:** The set of desired bearing vectors  $\Gamma$  is feasible. The desired bearing vectors of each agent  $i \in \mathcal{V}_f$ , given by  $\mathbf{g}_{ij}^*$ ,  $\forall j \in \mathcal{N}_i$ , are not all parallel to each other.

In this letter, the following problems will be studied.

**Problem 1:** Let Assumptions 1–3 hold. Design control laws for the followers modeled by (2) using only bearing vector measurements so that  $\mathbf{g}_{ij} \rightarrow \mathbf{g}_{ij}^*$ ,  $\forall (i, j) \in \mathcal{E}$  in finite time.

**Problem 2:** Suppose that Assumptions 1–3 hold. Further, suppose that if  $(i, j) \in \mathcal{E}$ , agent  $j$  sends its position estimate  $\hat{\mathbf{p}}_j$  to agent  $i$  and the leaders have information on their true positions. Design control laws for the followers modeled by (2) so that  $\mathbf{g}_{ij} \rightarrow \mathbf{g}_{ij}^*$ ,  $\forall (i, j) \in \mathcal{E}$  in finite time.

## III. FINITE-TIME BEARING-ONLY FORMATION MANEUVER

### A. Proposed Control Law

The following bearing-only control law is proposed to solve Problem 1:

$$\begin{aligned} \mathbf{u}_i = & -k_{i1} \sum_{j \in \mathcal{N}_i} \mathbf{P}_{\mathbf{g}_{ij}} \text{sig}(\mathbf{P}_{\mathbf{g}_{ij}} \mathbf{g}_{ij}^*)^\alpha \\ & - k_{i2} \beta \sum_{j \in \mathcal{N}_i} \mathbf{P}_{\mathbf{g}_{ij}} \text{sign}(\mathbf{P}_{\mathbf{g}_{ij}} \mathbf{g}_{ij}^*), \end{aligned} \quad (3)$$

where  $i \in \mathcal{V}_f$ ,  $\alpha \in (0, 1)$ ,  $\mathbf{M}_i = \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}$ ,  $k_{i1} = \lambda_1(\mathbf{M}_i)^{-\frac{\alpha+1}{2}}$ ,  $k_{i2} = \lambda_1(\mathbf{M}_i)^{-\frac{1}{2}}$ . In (3), the functions  $\text{sig}(\cdot)$  and  $\text{sign}(\cdot)$  are defined component-wise,  $\mathbf{P}_{g_{ij}} = \mathbf{I}_d - \mathbf{g}_{ij} \mathbf{g}_{ij}^\top \in \mathbb{R}^{d \times d}$  is the projection matrix associating with  $\mathbf{g}_{ij}$ .

Some remarks on the proposed control law (3) are given as follows. First,  $\mathbf{u}_i$  consists of two terms: the first term is for controlling the bearing-vectors of the agents to the desired ones; the second term is included to reject the disturbances resulting from leaders' motions which vary the bearing vectors. Second, as the leaders are not collinear,  $\mathbf{M}_i$  is symmetric positive definite [9]. One has  $\lambda_1(\mathbf{M}_i) \leq \lambda_n(\mathbf{M}_i) = \|\mathbf{M}_i\| = \|\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}\| \leq \sum_{j \in \mathcal{N}_i} \|\mathbf{P}_{g_{ij}}\| = |\mathcal{N}_i|$ . Thus  $k_{i1} \in (|\mathcal{N}_i|^{-\frac{\alpha+1}{2}}, \infty)$  and  $k_{i2} \in (|\mathcal{N}_i|^{-\frac{1}{2}}, \infty)$ . The number of neighbors for each follower  $i$  is at least three, so that if its neighbors are in a non-collinear configuration and no collision happens between agents,  $\lambda_1(\mathbf{M}_i) > 0$ .

## B. Convergence Analysis

Since  $\mathcal{G}$  has a directed acyclic structure, the convergence analysis begins from the first follower. It will be shown that the first follower can achieve the desired bearing vectors and follow the leaders in finite time. Then, finite-time convergence of the overall desired formation will be established based on mathematical induction.

**1) The First Follower:** Consider agent  $i = l+1$  moving under the control law (3). Define the subformation  $(\mathcal{K}_l, \mathbf{p}^l)$ , where  $\mathcal{K}_l$  is the complete graph of  $l$  vertices. From Assumption 1,  $\mathbf{v}^l = \mathbf{1}_l \otimes \mathbf{v}^*$  belongs to the space of infinitesimal bearing rigid motions of  $(\mathcal{K}_l, \mathbf{p}^l)$ . Thus, the relative bearing vector between the leaders are maintained during maneuver.

The desired position of follower  $i$  is determined by equations  $\mathbf{P}_{g_{ij}}^* (\mathbf{p}_i^* - \mathbf{p}_j) = \mathbf{0}$ ,  $\forall j \in \mathcal{N}_i$ . By summing up these equations, it follows that  $\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^* (\mathbf{p}_i^* - \mathbf{p}_j) = \mathbf{0}$  or equivalently  $\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^* \mathbf{p}_i^* = \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^* \mathbf{p}_j$ . According to the Assumption 1, the leaders are not colinear, which implies that the matrix  $\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^*$  is positive definite. It follows that  $\mathbf{p}_i^* = (\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^*)^{-1} (\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^* \mathbf{p}_j)$  and thus,  $\dot{\mathbf{p}}_i^* = \mathbf{v}_i^* = (\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^*)^{-1} (\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^* \dot{\mathbf{p}}_j) = \mathbf{v}^*$ .

**Lemma 1:** Suppose that Assumptions 1–3 hold. Under the control law (3), for  $i = l+1$ ,  $\mathbf{p}_i \rightarrow \mathbf{p}_i^*$  in finite time.

*Proof:* Consider the Lyapunov function  $V_i = \frac{1}{2} \|\mathbf{p}_i - \mathbf{p}_i^*\|^2$ , which is positive definite and radially unbounded. As the control law (3) is discontinuous, solution of (2) is understood in Filippov's sense [27]. One has  $\dot{V}_i \in^{a.e.} \dot{V}_i = \bigcap_{\xi \in \nabla V_i(\mathbf{p}_i)} \xi^\top K[\dot{\mathbf{p}}_i]$ . Then,  $\nabla V_i(\mathbf{p}_i) = \mathbf{p}_i - \mathbf{p}_i^*$ , and

$$\begin{aligned} \dot{V}_i &= (\mathbf{p}_i - \mathbf{p}_i^*)^\top \left( -k_{i1} \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} \text{sig}(\mathbf{P}_{g_{ij}} \mathbf{g}_{ij}^*)^\alpha \right. \\ &\quad \left. - k_{i2} \beta \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} K[\text{sign}](\mathbf{P}_{g_{ij}} \mathbf{g}_{ij}^*) - \mathbf{v}^* \right) \\ &= (\mathbf{p}_i - \mathbf{p}_i^*)^\top \left( -k_{i1} \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} \text{sig} \left( \mathbf{P}_{g_{ij}} \frac{\mathbf{p}_j - \mathbf{p}_i^*}{\|\mathbf{z}_{ij}^*\|} \right)^\alpha \right. \\ &\quad \left. - k_{i2} \beta \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} K[\text{sign}](\mathbf{P}_{g_{ij}} \mathbf{g}_{ij}^*) - \mathbf{v}^* \right). \end{aligned}$$

Because  $\mathbf{P}_{g_{ij}} (\mathbf{p}_j - \mathbf{p}_i^*) = \mathbf{P}_{g_{ij}} (\mathbf{p}_j - \mathbf{p}_i + \mathbf{p}_i - \mathbf{p}_i^*) = \mathbf{P}_{g_{ij}} (\mathbf{p}_i - \mathbf{p}_i^*)$ , it follows that

$$\begin{aligned} \dot{V}_i &= -k_{i1} (\mathbf{p}_i - \mathbf{p}_i^*)^\top \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} \text{sig} \left( \frac{\mathbf{P}_{g_{ij}} (\mathbf{p}_i - \mathbf{p}_i^*)}{\|\mathbf{z}_{ij}^*\|} \right)^\alpha \\ &\quad - k_{i2} \beta (\mathbf{p}_i - \mathbf{p}_i^*)^\top \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} K[\text{sign}](\mathbf{P}_{g_{ij}} \mathbf{g}_{ij}^*) \\ &\quad - (\mathbf{p}_i - \mathbf{p}_i^*)^\top \mathbf{v}^* \\ &\leq -k_{i1} (\mathbf{p}_i - \mathbf{p}_i^*)^\top \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} \text{sig} \left( \frac{1}{\|\mathbf{z}_{ij}^*\|} \mathbf{P}_{g_{ij}} (\mathbf{p}_i - \mathbf{p}_i^*) \right)^\alpha \\ &\quad - k_{i2} \beta (\mathbf{p}_i - \mathbf{p}_i^*)^\top \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} K[\text{sign}](\mathbf{P}_{g_{ij}} \mathbf{g}_{ij}^*) \\ &\quad + \|\mathbf{p}_i - \mathbf{p}_i^*\| \|\mathbf{v}^*\| \end{aligned} \quad (4)$$

From [27] that for any  $x \in \mathbb{R}$ ,  $xK[\text{sign}](x) = \{|x|\}$ , one has

$$\begin{aligned} &(\mathbf{p}_i - \mathbf{p}_i^*)^\top \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} \text{sig} \left( \frac{1}{\|\mathbf{z}_{ij}^*\|} \mathbf{P}_{g_{ij}} (\mathbf{p}_i - \mathbf{p}_i^*) \right)^\alpha \\ &= \sum_{j \in \mathcal{N}_i} \frac{1}{\|\mathbf{z}_{ij}^*\|^\alpha} |\mathbf{P}_{g_{ij}} (\mathbf{p}_i - \mathbf{p}_i^*)|^\alpha \\ &= \sum_{j \in \mathcal{N}_i} \frac{1}{\|\mathbf{z}_{ij}^*\|^\alpha} |(\mathbf{p}_i - \mathbf{p}_i^*)^\top \mathbf{P}_{g_{ij}} (\mathbf{p}_i - \mathbf{p}_i^*)|^{\frac{\alpha+1}{2}} \\ &\geq \frac{1}{\max_{j \in \mathcal{N}_i} (\|\mathbf{z}_{ij}^*\|)^\alpha} |(\mathbf{p}_i - \mathbf{p}_i^*)^\top \mathbf{M}_i (\mathbf{p}_i - \mathbf{p}_i^*)|^{\frac{\alpha+1}{2}} \\ &\geq \frac{1}{\max_{j \in \mathcal{N}_i} (\|\mathbf{z}_{ij}^*\|)^\alpha} (\lambda_1(\mathbf{M}_i))^{\frac{\alpha+1}{2}} \|\mathbf{p}_i - \mathbf{p}_i^*\|^{\alpha+1}, \end{aligned}$$

where the first inequality follows from [28, Lemma 2], and

$$\begin{aligned} &(\mathbf{p}_i - \mathbf{p}_i^*)^\top \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} K[\text{sign}](\mathbf{P}_{g_{ij}} \mathbf{g}_{ij}^*) \\ &= \sum_{j \in \mathcal{N}_i} (\mathbf{p}_i - \mathbf{p}_i^*)^\top \mathbf{P}_{g_{ij}} K[\text{sign}](\mathbf{P}_{g_{ij}} (\mathbf{p}_i - \mathbf{p}_i^*)) \\ &= \sum_{j \in \mathcal{N}_i} \|\mathbf{P}_{g_{ij}} (\mathbf{p}_i - \mathbf{p}_i^*)\|_1 \geq \left\| \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} (\mathbf{p}_i - \mathbf{p}_i^*) \right\|_1 \\ &\geq \left\| \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}} (\mathbf{p}_i - \mathbf{p}_i^*) \right\| \geq \lambda_1(\mathbf{M}_i)^{-\frac{1}{2}} \|\mathbf{p}_i - \mathbf{p}_i^*\|. \end{aligned}$$

Thus, one has

$$\dot{V}_i \leq -\eta V_i^{\frac{\alpha+1}{2}} - (\beta - \|\mathbf{v}^*\|) \|\mathbf{p}_i - \mathbf{p}_i^*\|, \quad (5)$$

where  $\eta = \frac{2^{\frac{\alpha+1}{2}}}{\max_{j \in \mathcal{N}_i} (\|\mathbf{z}_{ij}^*\|)^\alpha}$ . Based on [29],  $\mathbf{p}_i \rightarrow \mathbf{p}_i^*$  in a finite time upper bounded by  $T_{l+1} = \frac{2V_{l+1}(0)^{\frac{1-\alpha}{2}}}{\eta(1-\alpha)}$ . ■

**2) The  $n$ -Agent Formation:** For each agent  $i \geq l+1$ , the velocities of agents  $j \in \mathcal{N}_i \subset \{1, \dots, i-1\}$  are considered as external inputs to the dynamics of agent  $i$ . The convergence of the overall formation is given in the following theorem.

**Theorem 1:** Suppose that Assumptions 1–3 hold. Further, suppose that each agent and its neighbors are always non-collinear. Under control law (3),  $\mathbf{p}_i(t) \rightarrow \mathbf{p}_i^*$  in finite time

upper bounded by  $T_i = T_{i-1} + \frac{2V_i(T_{i-1})^{\frac{1-\alpha}{2}}}{\eta(1-\alpha)}$ ,  $i = l+1, \dots, n$  and  $T_l = 0$ , and  $\mathbf{u}_i$  is bounded  $\forall i = l+1, \dots, n$ .

*Proof:* We show this theorem by mathematical induction on  $i$ . From Lemma 1, the claim is true for  $i = l+1$ . Next, suppose that the claim holds until  $i$  ( $l+1 \leq i \leq n$ ), then  $\forall j \in \mathcal{N}_i$ , during  $[0, T_{j-1}]$ ,  $\mathbf{u}_j$  is bounded and  $\mathbf{p}_j = \mathbf{p}_j^*$  for  $t \geq T_j$ , where  $T_k = T_{k-1} + \frac{2V_i(T_{k-1})^{\frac{1-\alpha}{2}}}{\eta(1-\alpha)}$ ,  $\forall k = l+1, \dots, i-1$  and  $T_l = 0$ . Since agent  $i$  and its neighbor agents are not collinear and no collisions happen,  $\mathbf{M}_i = \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^*$  is always positive definite and  $k_{i1}, k_{i2} < \infty$  in  $t \in [0, T_{i-1}]$ . It follows that  $\dot{\hat{\mathbf{p}}}_i(t)$  is bounded and thus  $\|\mathbf{p}_i(T_{i-1}) - \mathbf{p}_i^*(T_{i-1})\| < \infty$ .

Next, for  $t \geq T_{i-1}$ , consider the Lyapunov function  $V_i = \frac{1}{2}\|\mathbf{p}_i - \mathbf{p}_i^*\|^2$ . Similar to Lemma 1, one has

$$\dot{V}_i \leq -\frac{\|\mathbf{p}_i - \mathbf{p}_i^*\|^{\alpha+1}}{\max_{j \in \mathcal{N}_i} \|\mathbf{z}_{ij}^*\|^\alpha} - \|\mathbf{p}_i - \mathbf{p}_i^*\|(\beta - \|\mathbf{v}^*\|). \quad (6)$$

This means  $\mathbf{p}_i(t) \rightarrow \mathbf{p}_i^*$  in a finite time upper bounded by  $\sum_{k=l+1}^i T_k$ , where  $T_i = \frac{2V_i(T_{i-1})^{\frac{1-\alpha}{2}}}{\eta(1-\alpha)}$ .

Therefore, the claim is also true for  $i = n$  by mathematical induction.  $\blacksquare$

*Remark 1:* After the target formation was achieved, the leaders can rescale the formation by adopting the reference velocity  $\mathbf{v}^l(t) \in \text{im}(\mathbf{1}_l \otimes \mathbf{I}_d, \mathbf{p}^l(t))$ ,  $\mathbf{v}^l(t) = \mathbf{v}_{\text{trans}}^l(t) + k_{\text{scale}}(t)\mathbf{v}_{\text{scale}}^l(t)$ , where  $\mathbf{v}_{\text{trans}}^l(t) = \mathbf{1}_l \otimes \mathbf{v}^*$  and  $\mathbf{v}_{\text{scale}}^l(t) = \frac{\mathbf{p}^l - \hat{\mathbf{p}}^l \otimes \mathbf{1}_l}{\|\mathbf{p}^l - \hat{\mathbf{p}}^l \otimes \mathbf{1}_l\|}$ ,  $\hat{\mathbf{p}}^l = (\mathbf{1}_l \otimes \mathbf{I}_d)\mathbf{p}^l/l$ , are the translational and scaling motions of the formation  $(\mathcal{K}_l, \mathbf{p}^l)$ , respectively. Suppose that  $\mathbf{v}_{\text{trans}}^l$  is bounded and  $k_{\text{scale}}(t) = \xi > 0$ ,  $t \in [t_1, t_2]$ ,  $0 \leq t_1 < t_2 < \infty$ , and  $k_{\text{scale}}(t) = 0$ , otherwise. This assumption describes that the formation mainly moves forward and the rescaling process is occasionally performed during the maneuver, e.g., before and after traversing a narrow alley. For  $t \geq t_2$ , convergence of the desired formation can be shown as in Theorem 1.

*Remark 2:* Since collision between agents and collinearity of  $\{\mathbf{p}_j\}_{j \in \mathcal{N}_i}$ ,  $\forall i = l+1, \dots, n$  are excluded, one cannot conclude in Theorem 1 about globally asymptotic stability of the target formation. Also, the mathematical induction cannot be established without finite time convergence of each follower since boundedness of  $\|\mathbf{p}_i - \mathbf{p}_i^*\|$ ,  $\forall t$  will not be guaranteed.

#### IV. FINITE-TIME FORMATION MANEUVER VIA TARGET POINT LOCALIZATION

##### A. Proposed Control Law

In this section, we propose an control strategy to solve Problem 2. Instead of directly controlling the position based on the bearing errors, each follower estimates its desired position with regard to the leaders and track that point. The following finite-time control law is proposed:

$$\begin{aligned} \dot{\hat{\mathbf{p}}}_i &= \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^* \text{sig}(\mathbf{P}_{g_{ij}}^*(\hat{\mathbf{p}}_j - \hat{\mathbf{p}}_i))^\alpha \\ &\quad + \gamma_i \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^* \text{sign}(\mathbf{P}_{g_{ij}}^*(\hat{\mathbf{p}}_j - \hat{\mathbf{p}}_i)) \end{aligned} \quad (7)$$

$$\dot{\mathbf{p}}_i = -\text{sig}(\mathbf{p}_i - \hat{\mathbf{p}}_i)^\alpha - \beta \text{sign}(\mathbf{p}_i - \hat{\mathbf{p}}_i), \quad (8)$$

where  $\alpha \in (0, 1)$  and  $\gamma_i \geq \beta \|\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^*\|^{-1}$ . Due to the directed acyclic structure of the graph  $\mathcal{G}$ , agent  $i$  can also

calculate the target point directly from the information by  $\mathbf{p}_i^* = (\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^*)^{-1} \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^* \mathbf{p}_j^*$ , and track that point under the control law (8). In case the beacons are stationary, a fixed-time network localization law was proposed in [30].

##### B. Convergence Analysis

Consider the first follower  $i = l+1$ . The following lemma will be proved.

*Lemma 2:* Let the assumptions of Problem 2 hold, for  $i = l+1$ ,  $\hat{\mathbf{p}}_i(t) \rightarrow \mathbf{p}_i^*$  and  $\mathbf{p}_i(t) \rightarrow \mathbf{p}_i^*$  in finite time.

*Proof:* First, consider the estimation dynamics (7). Using the Lyapunov function  $V = \frac{1}{2}\|\hat{\mathbf{p}}_i - \mathbf{p}_i^*\|^2$  and keeping in mind that  $\hat{\mathbf{p}}_j = \mathbf{p}_j^*$ ,  $\forall j \in \mathcal{N}_i$ , one has

$$\begin{aligned} \dot{V} &= (\hat{\mathbf{p}}_i - \mathbf{p}_i^*)^\top \left( \mathbf{P}_{g_{ij}}^* \sum_{j \in \mathcal{N}_i} \text{sig}(\mathbf{P}_{g_{ij}}^*(\mathbf{p}_j^* - \mathbf{p}_i^* + \mathbf{p}_i^* - \hat{\mathbf{p}}_i))^\alpha \right. \\ &\quad \left. + \gamma_i \mathbf{P}_{g_{ij}}^* \sum_{j \in \mathcal{N}_i} \text{sign}(\mathbf{P}_{g_{ij}}^*(\mathbf{p}_j^* - \mathbf{p}_i^* + \mathbf{p}_i^* - \hat{\mathbf{p}}_i)) - \mathbf{v}^* \right) \\ &= - \sum_{j \in \mathcal{N}_i} |\mathbf{P}_{g_{ij}}^*(\mathbf{p}_i^* - \hat{\mathbf{p}}_i)|^{\alpha+1} \\ &\quad - \gamma_i \sum_{j \in \mathcal{N}_i} \|\mathbf{P}_{g_{ij}}^*(\mathbf{p}_i^* - \hat{\mathbf{p}}_i)\|_1 - (\hat{\mathbf{p}}_i - \mathbf{p}_i^*)^\top \mathbf{v}^* \\ &\leq - \left| (\mathbf{p}_i^* - \hat{\mathbf{p}}_i)^\top \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^*(\mathbf{p}_i^* - \hat{\mathbf{p}}_i) \right|^{\frac{\alpha+1}{2}} \\ &\quad - \gamma_i \left\| \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^*(\mathbf{p}_i^* - \hat{\mathbf{p}}_i) \right\|_1 + \|\mathbf{v}^*\| \|\mathbf{p}_i^* - \hat{\mathbf{p}}_i\| \\ &\leq -\eta_i |(\mathbf{p}_i^* - \hat{\mathbf{p}}_i)^\top (\mathbf{p}_i^* - \hat{\mathbf{p}}_i)|^{\frac{\alpha+1}{2}} \\ &\quad - \gamma_i \left\| \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^* \right\| \|\mathbf{p}_i^* - \hat{\mathbf{p}}_i\| + \|\mathbf{v}^*\| \|\mathbf{p}_i^* - \hat{\mathbf{p}}_i\| \\ &\leq -\chi_i V^{\frac{\alpha+1}{2}} - \left( \gamma_i \|\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^*\| - \|\mathbf{v}^*\| \right) \|\mathbf{p}_i^* - \hat{\mathbf{p}}_i\| \end{aligned} \quad (9)$$

where  $\eta_i = \lambda_1(\sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}}^*)^{\frac{\alpha+1}{2}}$  and  $\chi_i = \eta_i 2^{\frac{\alpha+1}{2}}$ . It follows that  $\hat{\mathbf{p}}_i(t) \rightarrow \mathbf{p}_i^*$  in finite time upper bounded by  $T_i$ , and  $\hat{\mathbf{v}}_i \rightarrow \mathbf{v}_i^*$  in finite time.

Next, consider the position tracking control law (8) with the Lyapunov function  $W = \frac{1}{2}\|\mathbf{p}_i - \hat{\mathbf{p}}_i\|^2$ . For  $t \leq T_i$ ,

$$\begin{aligned} \dot{W} &= -(\mathbf{p}_i - \hat{\mathbf{p}}_i)^\top \text{sig}(\mathbf{p}_i - \hat{\mathbf{p}}_i)^\alpha \\ &\quad - \beta (\mathbf{p}_i - \hat{\mathbf{p}}_i)^\top \text{sign}(\mathbf{p}_i - \hat{\mathbf{p}}_i) - (\mathbf{p}_i - \hat{\mathbf{p}}_i)^\top \hat{\mathbf{v}}_i \\ &\leq -\|\mathbf{p}_i - \hat{\mathbf{p}}_i\|^{\alpha+1} - \beta \|\mathbf{p}_i - \hat{\mathbf{p}}_i\|_1 + \|\mathbf{p}_i - \hat{\mathbf{p}}_i\| \|\hat{\mathbf{v}}_i\|, \end{aligned} \quad (10)$$

which shows that  $\|\mathbf{p}_i - \hat{\mathbf{p}}_i\|$  is globally ultimately bounded. Together with the boundedness of  $\|\hat{\mathbf{p}}_i - \mathbf{p}_i^*\|$ , it follows from the triangle inequality that  $\|\mathbf{p}_i - \mathbf{p}_i^*\| \leq \|\mathbf{p}_i - \hat{\mathbf{p}}_i\| + \|\hat{\mathbf{p}}_i - \mathbf{p}_i^*\|$ , i.e.,  $\|\mathbf{p}_i - \mathbf{p}_i^*\|$  is also bounded.

Now, for  $t \geq T_i$ ,  $\hat{\mathbf{p}}_i = \mathbf{p}_i^*$  and  $\|\hat{\mathbf{v}}_i\| = \|\mathbf{v}^*\| \leq \beta$ , consider the function  $W_1 = \frac{1}{2}\|\mathbf{p}_i - \hat{\mathbf{p}}_i^*\|^2$ . Similar to (10), one finds that  $\dot{W}_1 \leq -\|\mathbf{p}_i - \hat{\mathbf{p}}_i\|^{\alpha+1} - \|\mathbf{p}_i - \hat{\mathbf{p}}_i\|(\beta - \|\hat{\mathbf{v}}_i\|) \leq -\zeta_i W_1^{\frac{\alpha+1}{2}}$ , where  $\zeta_i = 2^{\frac{\alpha+1}{2}} > 0$  is a positive constant. Thus,  $\mathbf{p}_i \rightarrow \mathbf{p}_i^*$  in finite time.  $\blacksquare$

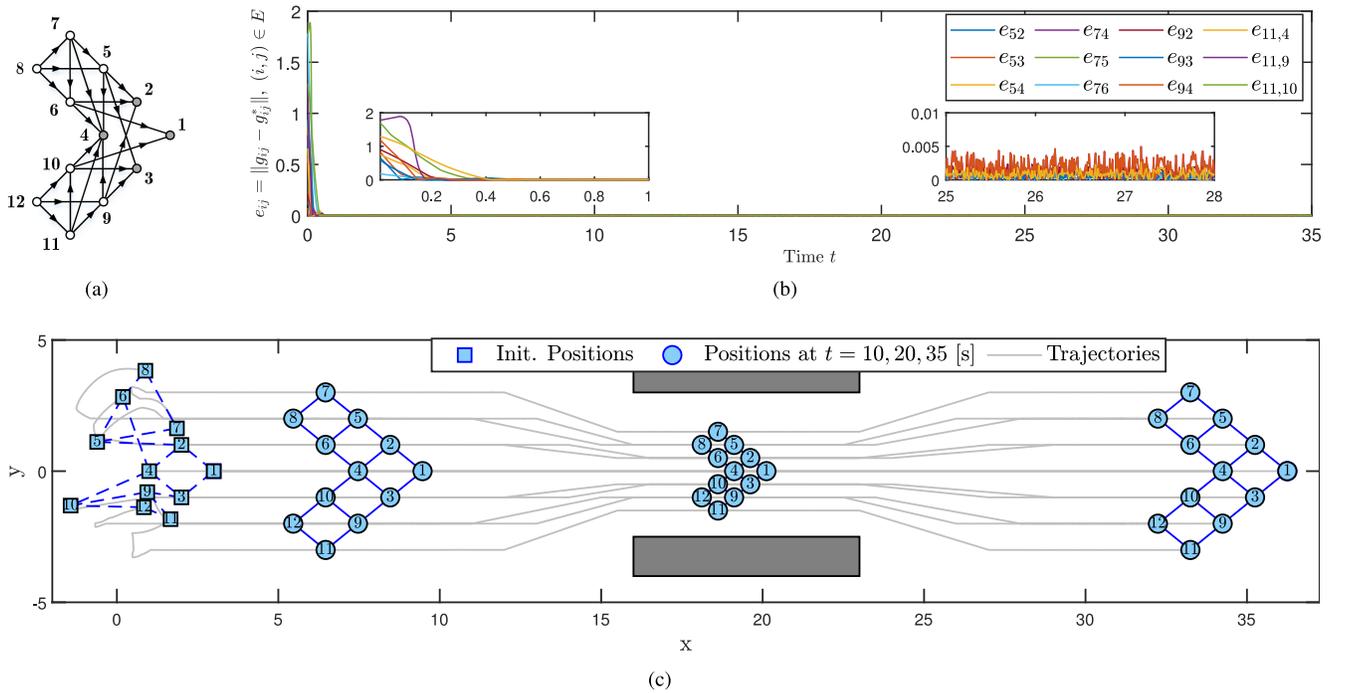


Fig. 1. Simulation 1: (a) The acyclic leader-follower graph  $\mathcal{G}$ ; (b) Bearing errors vs time [s]; (c) Trajectory of the agents.

*Theorem 2:* Under the control laws (7)–(8), the desired moving formation is achieved in finite time.

*Proof:* The proof follows from Lemma 2 and mathematical induction on  $i = l + 1$  to  $n$ . ■

*Remark 3:* Observe that (8) steers the agent to the virtual target along a curve. Let  $d = 2$  and assume that the agents are equipped with proximity sensors which can sense the distance to an obstacle located at  $\mathbf{p}_o$  within a small range  $d_{max}$ . To avoid collision with an obstacle, for  $0 < d < d_{max}$ , (8) can be modified as follows:

$$\mathbf{u}_i = -(1 - \zeta) \text{sig}(\mathbf{p}_i - \hat{\mathbf{p}}_i)^\alpha + \beta \text{sign}(\mathbf{p}_i - \hat{\mathbf{p}}_i) + \zeta \left( k(\mathbf{p}_i - \mathbf{p}_o) + \mathbf{g}_{io}^\perp \right), \quad (11)$$

where  $\zeta = 1$  if  $\|\mathbf{p}_i - \mathbf{p}_o\| < d$  and  $\zeta = 0$  if  $\|\mathbf{p}_i - \mathbf{p}_o\| \geq d$ ,  $\mathbf{g}_{io}^\perp = \mathbf{J} \frac{\mathbf{p}_o - \mathbf{p}_i}{\|\mathbf{p}_o - \mathbf{p}_i\|}$ ,  $\mathbf{J} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ ,  $k > 1$  is a sufficiently large control gain.

*Remark 4:* A fixed-time position estimation dynamics can be designed based on (7) and [30], [31] as follows:

$$\begin{aligned} \dot{\hat{\mathbf{p}}}_i &= \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}^*} \left( \text{sig}(\mathbf{P}_{g_{ij}^*}(\hat{\mathbf{p}}_j - \hat{\mathbf{p}}_i))^\alpha + \text{sig}(\mathbf{P}_{g_{ij}^*}(\hat{\mathbf{p}}_j - \hat{\mathbf{p}}_i))^\rho \right) \\ &+ \gamma_i \sum_{j \in \mathcal{N}_i} \mathbf{P}_{g_{ij}^*} \text{sign}(\mathbf{P}_{g_{ij}^*}(\hat{\mathbf{p}}_j - \hat{\mathbf{p}}_i)), \end{aligned} \quad (12)$$

where  $0 < \alpha < 1$  and  $\rho > 1$ . Note that (12) is also applicable for infinitesimally bearing rigid leader-follower formations.

## V. SIMULATION RESULTS

### A. Simulation 1: Bearing-Only Control Law

Consider a formation of 12 agents with graph as shown in Fig. 1(a). Agents 1–4 are leaders, which move with the reference velocity  $\mathbf{v}^l$  given as follows

- For  $t \in [0, 10]$ ,  $\mathbf{v}^l = \mathbf{1}_4 \otimes \mathbf{f}_1$ , where  $\mathbf{f}_1 = [1.9 - 0.14t, 0]^\top$ . Leaders move in straight lines along the  $x$ -axis. At  $t = 10$ ,  $\mathbf{v}_1(10) = [0.5, 0]^\top$ .
- For  $t \in [10, 15]$ ,  $\mathbf{v}^l = \mathbf{1}_4 \otimes \mathbf{f}_2 - \frac{\mathbf{h}}{5\|\mathbf{h}\|}$ , where  $\mathbf{f}_2 = [0.5, 0]^\top$ ,  $\mathbf{h} = \mathbf{p}^l - \mathbf{1}_4 \otimes \bar{\mathbf{p}}^l$ , and  $\bar{\mathbf{p}}^l = \frac{1}{4}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \mathbf{p}_4)$  is the geometric center of four leaders. Leaders go along the  $x$ -axis and downscale the formation's size to fit the alley. At  $t = 15$ s,  $\mathbf{v}_1(15) = [0.5, 0]^\top$ .
- For  $t \in [15, 25]$ ,  $\mathbf{v}^l = \mathbf{1}_4 \otimes \mathbf{f}_3$ , where  $\mathbf{f}_3 = [0.5 + 0.05(t - 15), 0]^\top$ . Leaders move through the alley. At  $t = 25$ s,  $\mathbf{v}_1(25) = [1, 0]^\top$ .
- For  $t \in [25, 30]$ ,  $\mathbf{v}^l = \mathbf{1}_4 \otimes \mathbf{f}_4 + \frac{\mathbf{h}}{5\|\mathbf{h}\|}$ , where  $\mathbf{f}_4 = [1, 0]^\top$ . The formation has passed the alley. Leaders go along the  $x$ -axis and upscale the formation's size back to normal. At  $t = 30$ s,  $\mathbf{v}_1(30) = [1, 0]^\top$ .
- For  $t \in [30, 35]$ ,  $\mathbf{v}^l = \mathbf{1}_4 \otimes \mathbf{f}_5$ , where  $\mathbf{f}_5 = [1 + 0.1(t - 30), 0]^\top$ . The formation accelerates and continues to move forward along the  $x$ -axis.

The followers adopt the control law (3) with  $\beta = 2$ ,  $\alpha = 0.5$ . The simulation results are depicted in Fig. 1(b), (c). The target formation shape is achieved in less than 1 second (see Fig. 1(b)) and maintained except when the formation rescales its size. Thus, simulation result is consistent with Theorem 1.

### B. Simulation 2: Target Point Localization-Based Control Law

The same 12-agent formation is simulated under the control law (7)–(8) for 5 seconds. The initial estimates are randomly selected. The trajectories of the agents are depicted in Fig. 2(a). The desired formation is achieved in less than 1 second, which is consistent with Theorem 2.

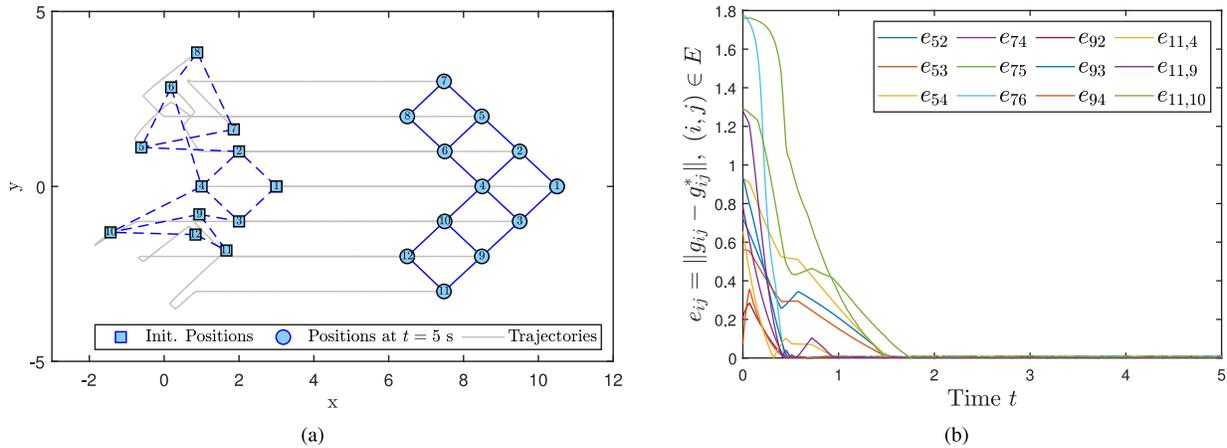


Fig. 2. Simulation 2: (a) Trajectory of the agents. (b) Bearing errors vs time [s].

## VI. CONCLUSION

In this letter, two finite-time bearing-based tracking control laws for acyclic leader-follower formations have been proposed. The analysis partially explains how individuals can follow leaders, who are moving at a time-varying velocity, in collective behaviors such as bird immigration. As suggested in [23], it will be interesting to consider the problem with delay and switching in sensing/communication, or when agents can measure only the subtended bearing angles.

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