



# Cross-phase modulation between pulse matched lights in optical loop system

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## ABSTRACT

We observed conditional cross-phase modulation (C-XPM) applicable to quantum phase gate. Combinations of two probe field's polarizations conditioned XPM of two probe fields in the corresponding nonlinear optical system. With optimized parameters of multi-photon transition, we could increase C-XPM more than 1 rad. with small probe absorption. Group velocity matching ratio between probe pulses was estimated to 1.2 in the optical loop system. Because multi-photon interference depends on the ratio between the Rabi frequencies of the probe fields, not the individual intensity, we expect the same phase shift between few photons level's probe pulses in our scheme.

## 1. Introduction

Controlling a photon using another photon, e.g., cross-phase modulation (XPM) or all-optical switching (AOS), would be an intriguing and important scheme in quantum optical applications [1,2]. In such a scheme, a large phase shift with small loss and efficient  $0 \leftrightarrow 1$  switching are core techniques. However, it is quite difficult to realize these techniques since the interactions between the control and target photons are too weak. Thus, some ideas have been proposed and developed to overcome this fundamental problem. Using cavity quantum electrodynamics is one of the powerful method to obtain a strong photon-photon interaction [3–5]. Recently, J. Volz et al. [6] demonstrated almost  $\pi$ -phase shift with whispering gallery-mode resonator interfaced by Rb atom presence optical nanofiber and Fushman et al. [7] showed  $\pi/4$ -phase shift with a single quantum dot coupled to a photonic crystal nano-cavity. On the other hand, non-cavity coupled systems which have better characteristics for deterministic measurements, such as electromagnetically induced transparency (EIT) based system [8–10] and hollow fiber containing atoms [11] also have obtained enhanced interactions between photons even though they have a weaker interaction strength than cavity-coupled systems. To enhance the phase shift in a single photon level with non-cavity coupled system, modified N-type systems, for example tripod [12], M-type [13–15] and gain assisted N-type system [16] were proposed. Some of those proposed schemes would achieve  $\pi$ -phase shift, however, phase shift per photon ( $\sim 10^{-6}$  rad.) is still not enough to be satisfied for application [17]. Meanwhile, 0.3 mrad phase shift per photon in Rb atoms confined hollow-core photonic bandgap fiber was achieved without cavity [18]. Recently, XPM using double- $\Lambda$  systems were reported in Ref. [19,20] where  $\pi$ -phase shift by changing relative phase was experimentally demonstrated, however the group velocities of two probe pulses could

not be matched because they used asymmetrically detuned double- $\Lambda$  system.

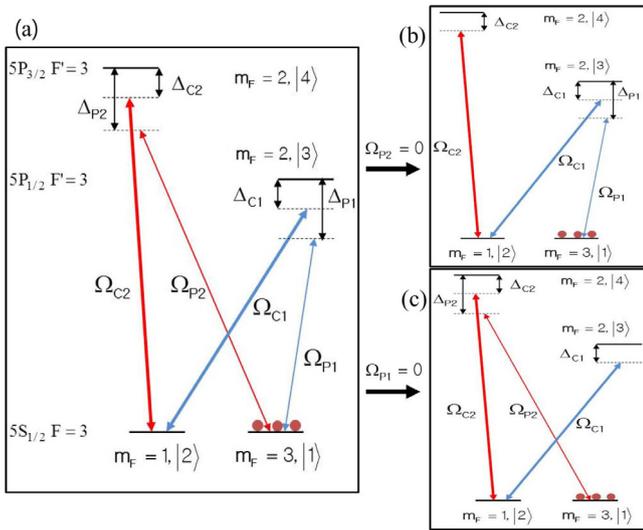
In our system, a few photon level of probe pulses (16 photons per pulse) induced C-XPM (will be defined below) in the symmetrically detuned double- $\Lambda$  system. The group velocity of two probe fields was theoretically estimated almost equivalent in the condition of the symmetry detuning. The group velocity matching between probe pulses is one of the most important qualities for the application of quantum logic gate. We experimentally demonstrated 2 rad. of C-XPM by conditioning polarization combination. The amount of phase shift was depending on optical depth, detuning of coupling fields, two-photon detuning, and coupling Rabi frequencies. The transparency in a four-level double- $\Lambda$  system was induced by multi-photon interference in an optical closed loop system which has a different mechanism from conventional EIT [21]. One of the advantages using a double- $\Lambda$  system is one can expect the same efficient of XPM between few photons' level of probe fields, because the characteristics of a double- $\Lambda$  system has dependence on the ratio, not the intensities, between Rabi frequencies of probe fields. Therefore, we can expect the same C-XPM less than our probe photon numbers 16.

## 2. Theory

A double-lambda system was shown in Fig. 1(a).  $\sigma^+$ -polarized two strong coupling fields  $\Omega_{C1}$  and  $\Omega_{C2}$  drive the  $|F = 3, m_F = 1\rangle$  ( $|2\rangle$ )  $\rightarrow |F' = 3, m_F = 2\rangle$  ( $|3\rangle, |4\rangle$ ) of  $^{85}\text{Rb}$   $D_1$  and  $D_2$  transition lines. Two  $\sigma^-$ -polarized weak probe fields  $\Omega_{P1}$  and  $\Omega_{P2}$  drive the  $|F = 3, m_F = 3\rangle$  ( $|1\rangle$ )  $\rightarrow |F' = 3, m_F = 2\rangle$  ( $|3\rangle, |4\rangle$ ) of  $^{85}\text{Rb}$   $D_1$  and  $D_2$  transition lines. Here,  $\Omega_{ij} = |\Omega_{ij}|e^{i\phi_{ij}}$  described amplitude  $|\Omega_{ij}|$  and phase  $e^{i\phi_{ij}}$  is Rabi frequencies of probe and coupling fields. Each coupling-probe pair

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**Fig. 1.** (a) A double- $\Lambda$  system implemented with Zeeman level of  $^{85}\text{Rb}$   $D_1$  and  $D_2$  transition level. The detuning of the coupling lasers  $\Omega_{C1}$  and  $\Omega_{C2}$  are denoted by  $\Delta_{C1}$  and  $\Delta_{C2}$  and the two-photon detuning  $\delta_i \equiv \Delta_{P_i} - \Delta_{C_i}$  ( $i = 1, 2$ ) where  $\Delta_{P_i}$  is the probe detuning. (b) Four-level N-type system.  $\Omega_{P2}$  was turned off (N1). (c) Four-level N-type system.  $\Omega_{P1}$  was turned off (N2).

$(\Omega_{C1} - \Omega_{P1}, \Omega_{C2} - \Omega_{P2})$  forms a  $\Lambda$ -type EIT system. The mechanism of a double- $\Lambda$  system could be analyzed by the interference between one-photon  $|1\rangle \rightarrow |3\rangle$  and three-photon excitation path  $|1\rangle \rightarrow |3\rangle \rightarrow |2\rangle \rightarrow |4\rangle$  (equivalent with  $|1\rangle \rightarrow |4\rangle$ ,  $|1\rangle \rightarrow |4\rangle \rightarrow |2\rangle \rightarrow |3\rangle$ ) [21]. Since this system has a phase dependence of the four fields ( $\phi_r = \phi_{P1} - \phi_{C1} + \phi_{C2} - \phi_{P2}$ , call this relative phase), one can control the transmission of probe fields by controlling the relative phase [22,23]. A double- $\Lambda$  system has been studied in a resonance ( $\Delta_{C1} = \Delta_{C2} = 0$ ). In this case, the transmission of two probe beams is maximized when the relative phase is 0 and decreased with the relative phase increasing. This resonant system, however, has a poor figure of merit for XPM which is defined by the ratio of the phase shift and absorption of probe field ( $\eta = \Delta\phi/\Delta\alpha$ ). Because coupling fields enhance the probe absorption when one probe field is extinguished, it decreases figure of merit. To avoid this problems, we adopt a non-resonant system where the coupling and probe fields are detuned from optical transition. There is a critical difference from resonant double- $\Lambda$  system, with changing the relative phase, the transmission of one probe beam is increasing while the other one is decreasing [24]. Here, we fixed the relative phase where the transmission of both probe fields are about 75% compared to input intensities. Following the notation in Ref. [13], our XPM could be written as,

$$|\sigma^\pm\rangle_{P1}|\sigma^\pm\rangle_{P2} \rightarrow e^{i\varphi_\pm^{P1} + i\varphi_\pm^{P2}} |\sigma^\pm\rangle_{P1}|\sigma^\pm\rangle_{P2}, \quad (1)$$

where  $\sigma^\pm$  is polarization of each probe field, and  $\phi^\pm$  is phase shift of  $\sigma^\pm$ -polarized probe fields. C-XPM  $\phi_m$  is written as,

$$\varphi_m = (\varphi_-^{P1} + \varphi_-^{P2}) - (\varphi_+^{P1} + \varphi_+^{P2}). \quad (2)$$

Here, C-XPM  $\phi_m$  means the difference of phase shift between polarization combinations. One might implement quantum logic gate with proper 'truth table' prepared by C-XPM [1,3]. If it is  $\pi$  between quantum bits, universal quantum gate can be implemented. In our system,  $|\sigma^-\rangle_{P1}|\sigma^-\rangle_{P2}$  establish a double- $\Lambda$  system in Fig. 1(a).  $|\sigma^-\rangle_{P1}|\sigma^+\rangle_{P2}$  and  $|\sigma^+\rangle_{P1}|\sigma^-\rangle_{P2}$  represent N-type system in Fig. 1(b) and (c), respectively, because there is no transition with  $\sigma^+$ -polarized probe fields.  $|\sigma^+\rangle_{P1, P2}$  corresponds to vacuum propagation of  $P1$  or  $P2$ . For the convenience, we will call  $|\sigma^-\rangle_{P1}|\sigma^-\rangle_{P2}$  as DL,  $|\sigma^-\rangle_{P1}|\sigma^+\rangle_{P2}$  as N1, and  $|\sigma^+\rangle_{P1}|\sigma^-\rangle_{P2}$  as N2 system through the paper. The Eq. (1) is for single photon Fock-state not coherent state. Our experiment and theory is not reached single photon quantum state in this work. However, our classical gate

might be expanded to single photon Fock-state for quantum computing in the future.

To discuss the system theoretically, we consider propagation behavior of the probe beams in the atomic medium. Because the coupling fields are much stronger than the probe beams ( $|\Omega_{C1}| \gg |\Omega_{P1}|$ ,  $|\Omega_{C2}| \gg |\Omega_{P2}|$ ), we can assume all the populations are on the state  $|1\rangle$ . The interaction between atomic medium and the applied fields can be described by a set of equations for the probability amplitudes and Maxwell-Schrödinger equations:

$$\begin{aligned} \dot{a}_2 &= i\Omega_{P1}^* a_3 + i\Omega_{P2}^* a_4 + i(\delta_1 + i\frac{\gamma_2}{2})a_2, \\ \dot{a}_3 &= i\Omega_{P1} + i\Omega_{C1} a_2 + i(\delta_2 + i\frac{\gamma_3}{2})a_3, \\ \dot{a}_4 &= i\Omega_{P2} + i\Omega_{C2} a_2 + i(\delta_3 + i\frac{\gamma_4}{2})a_4, \end{aligned} \quad (3)$$

$$\frac{\partial}{\partial z} \Omega_{P1} + \frac{1}{c} \frac{\partial}{\partial t} \Omega_{P1} = iK_{13} a_3,$$

$$\frac{\partial}{\partial z} \Omega_{P2} + \frac{1}{c} \frac{\partial}{\partial t} \Omega_{P2} = iK_{14} a_4, \quad (4)$$

where,  $\gamma_2$  is decoherence rate of the ground state  $|1\rangle$  and  $|2\rangle$ ,  $\gamma_3$  and  $\gamma_4$  are spontaneous decay rate of the excited state  $|3\rangle$  and  $|4\rangle$ , respectively and  $K_{1j} = 2\pi N \omega_{1j} |\mu_{1j}|^2 / (j = 3, 4)$ . The frequency detunings for the respective transition are defined as  $\delta_1 = \Delta_{P1} - \Delta_{C1}$ ,  $\delta_2 = \Delta_{P2} - \Delta_{C2}$ , and  $\delta_3 = \Delta_{P1} - \Delta_{C1} + \Delta_{C2}$ . With time derivative terms of Eqs. (3) being zero, the steady state solutions are obtained as,

$$\begin{aligned} a_2 &= \frac{D_3 \Omega_{C1}^*}{\Delta} \Omega_{P1} + \frac{D_2 \Omega_{C2}^*}{\Delta} \Omega_{P2}, \\ a_3 &= \frac{|\Omega_{C2}|^2 - D_1 D_3}{\Delta} \Omega_{P1} + \frac{\Omega_{C1} \Omega_{C2}^*}{\Delta} \Omega_{P2}, \\ a_4 &= \frac{|\Omega_{C1}|^2 - D_1 D_2}{\Delta} \Omega_{P2} + \frac{\Omega_{C1}^* \Omega_{C2}}{\Delta} \Omega_{P1}, \end{aligned} \quad (5)$$

where  $D_2 = \delta_2 + i\gamma_2/2$ ,  $D_1 = \delta_1 + i\gamma_3/2$ ,  $D_3 = \delta_3 + i\gamma_4/2$ ,  $\Delta = D_1 D_2 D_3 - D_3 |\Omega_{C1}|^2 - D_2 |\Omega_{C2}|^2$ . By substituting Eqs. (5) into Eqs. (6) with time derivation terms being zero, one can obtain

$$\begin{aligned} \frac{d}{dz} \Omega_{P1} &= iK_{13} \frac{|\Omega_{C2}|^2 - D_1 D_3}{\Delta} \Omega_{P1} - iK_{13} \frac{\Omega_{C1} \Omega_{C2}^*}{\Delta} \Omega_{P2}, \\ \frac{d}{dz} \Omega_{P2} &= iK_{14} \frac{|\Omega_{C1}|^2 - D_1 D_2}{\Delta} \Omega_{P2} - iK_{14} \frac{\Omega_{C2} \Omega_{C1}^*}{\Delta} \Omega_{P1}. \end{aligned} \quad (6)$$

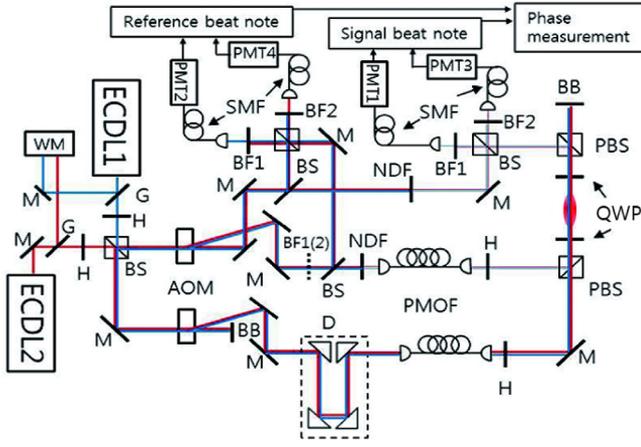
To obtain a time dependent solution for describing pulse propagation, we take a Fourier transform from Eqs. (3) and (4). And then, following the same process of finding a steady state solution, we can obtain a Fourier transformed time dependent solution written as,

$$\begin{aligned} W_{P1}(z, \omega) &= [(K_- e^{-i\zeta z} + K_+ e^{i\zeta z}) W_{P1}(0, \omega) \\ &+ J_3 (e^{i\zeta z} - e^{-i\zeta z}) W_{P1}(0, \omega)] \frac{e^{i\Theta z}}{2\zeta}, \end{aligned} \quad (7)$$

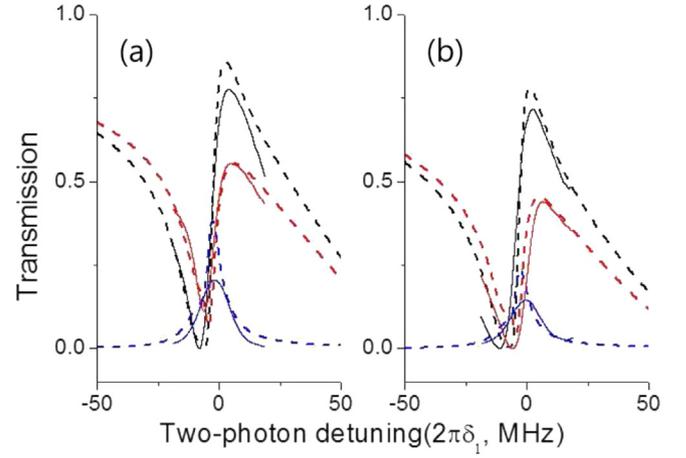
$$\begin{aligned} W_{P2}(z, \omega) &= [J_4 (e^{-i\zeta z} - e^{i\zeta z}) W_{P2}(0, \omega) + (K_+ e^{-i\zeta z} + K_- e^{i\zeta z}) \\ &\times W_{P2}(0, \omega)] \frac{e^{i\Theta z}}{2\zeta}, \end{aligned} \quad (8)$$

where  $\Theta(\omega) = \Theta = (J_1 + J_2)/2$ ,  $\zeta(\omega) = \zeta = \sqrt{U^2 + J_3 J_4}$ ,  $U(\omega) = U = (J_1 - J_2)/2$ ,  $K_\pm(\omega) = K_\pm = \pm U + \zeta$ ,  $J_{1(2)}(\omega) = J_{1(2)} = K_{12(13)} \xi_{1(2)} + \omega/c$ ,  $\xi_{1(2)} = (|\Omega_{C2(1)}|^2 - D_{1T} D_{3T(2T)})/\Delta_T$  and  $J_{3(4)}(\omega) = J_{3(4)} = K_{12(13)} (-\Omega_{C1(C2)} \Omega_{C2(C1)}^*)/\Delta_T$ . And  $W_{P1(P2)}$ ,  $D_{1T}$ ,  $D_{2T}$ ,  $D_{3T}$ , and  $\Delta_T$  are the Fourier transforms of  $\Omega_{P1(P2)}$ ,  $D_1$ ,  $D_2$ ,  $D_3$ , and  $\Delta$ , respectively. Finally, with approximation of ignoring the  $\Theta(\omega)$  terms except  $\Theta = (\Theta_0 + \Theta_{1\omega})$ , we obtained inverse Fourier transform

$$\begin{aligned} \Omega_{P1}(z, t) &= \left[ (K_+ e^{2i\zeta z} + K_-) \Omega_{P1} \left( 0, t - \frac{z}{v_{gP1}} \right) \right. \\ &+ J_3 (-1 + e^{2i\zeta z}) \Omega_{P2} \left( 0, t - \frac{z}{v_{gP1}} \right) \left. \right] \frac{e^{i(\Theta_0 - \zeta)z}}{2\zeta}, \end{aligned} \quad (9)$$



**Fig. 2.** Schematic of the experiment in a non-resonance double- $\Lambda$  system. ECDL1(2); 795 nm (780 nm) external cavity diode laser. M; mirror, HWP; half-wave plate, BS; beam splitter, AOM; acousto-optic modulator, BF1(2); 795 nm (780 nm) band-pass filter, D; delay line, NDF; neutral density filter, PMOF; polarization maintaining optical fiber, PBS; polarization beam splitter, QWP; quarter-wave plate, BB; beam blocker, SMF; single-mode fiber, PMT1(2); photomultiplier tube for  $\Omega_{02}(\Omega_{03})$ .



**Fig. 3.** Transmission of CW probe fields. (a) Transmission of P1 in a double- $\Lambda$  (black line), N1(P2 blocked)-system (red line). Blue line is its four wave mixing signal when P2 blocked. (b) Transmission of P2 in a double- $\Lambda$  (black line), N2(P1 blocked)-system (red line). Blue line is its four wave mixing signal when P1 blocked. Dashed lines are theoretical calculations. The parameters for the calculations are  $|\Omega_{C1}| = |\Omega_{C2}| = 0.6\gamma_3$ ,  $\gamma_1 = 0.02\gamma_3$ ,  $|\Omega_{P1}| = |\Omega_{P2}| = 0.002|\Omega_{C1}|$ , and  $N = 2.5 \times 10^9 \text{ cm}^{-3}$ .

$$\Omega_{gP2}(z, t) = \left[ J_4(-1 + e^{2i\zeta z}) \Omega_{P2}\left(0, t - \frac{z}{v_{gP2}}\right) + (K_- e^{2i\zeta z} + K_+) \Omega_{P1}\left(0, t - \frac{z}{v_{gP2}}\right) \right] \frac{e^{i(\theta_0 - \zeta)z}}{2\zeta}, \quad (10)$$

where the group velocities  $v_{gP1}$  and  $v_{gP2}$  are given as

$$v_{gP1} = \left\{ \frac{1}{c} + \frac{K_{13}}{2} \left[ \xi_1 \left( \xi_1 + \xi_2 - \frac{D_1 D_3}{\Delta} \right) - \frac{D_1 + D_3}{\Delta} \right] \right\}^{-1},$$

$$v_{gP2} = \left\{ \frac{1}{c} + \frac{K_{23}}{2} \left[ \xi_2 \left( \xi_1 + \xi_2 - \frac{D_1 D_2}{\Delta} \right) - \frac{D_1 + D_2}{\Delta} \right] \right\}^{-1}, \quad (11)$$

which is determined by  $1/\Theta$  (1). Here, because we assume  $\Delta_{P1} = \Delta_{P2}$ ,  $\Delta_{C1} = \Delta_{C2}$ ,  $|\Omega_{P1}| = |\Omega_{P2}|$ , and  $|\Omega_{C1}| = |\Omega_{C2}|$ , the ratio of the two group velocities is  $v_{gP1}/v_{gP2} \sim 1.2$  [25].

### 3. Experiment

We produced cold  $^{85}\text{Rb}$  atom cloud as an optical medium by magneto-optical trap(MOT). To generate a dense medium, we used temporal dark spontaneous force optical trap (SPOT) and rectangular-shaped anti-Helmholtz coils. Typically, the optical density (OD) 50 was estimated in our MOT system. Our experimental scheme is shown in Fig. 2.

To form a double- $\Lambda$  system, we used two external cavity diode laser (ECDL) systems. The laser frequency of ECDL1(2) corresponded to  $^{85}\text{Rb}$   $|5S_{1/2}, F=3\rangle \rightarrow |5P_{1/2}, F'=3\rangle$  ( $^{85}\text{Rb}$   $|5S_{1/2}, F=3\rangle \rightarrow |5P_{3/2}, F'=3\rangle$ ). To measure the frequencies of the laser system, we used wavelength meter (High Finesse, WSU2, 2MHz resolution) and to stabilize the frequencies, polarization spectroscopic systems were used. The two laser beams were overlapped by a beam splitter in front of the ECDLs. Output beams from the beam splitter were used as coupling and probe beams. Because all overlapped propagating beams have the opposite sign of the phase terms, arbitrary phase and external vibrations were canceled out [23,26]. In this case, the relative phase ( $\phi_r = \phi_{P1} - \phi_{C1} + \phi_{C2} - \phi_{P2}$ ) could be controlled by changing the length of the delay line of  $42 \mu\text{m}$  which correspond to half wavelength of separation between  $^{85}\text{Rb}$   $D_1$  and  $D_2$  transition line. Coupling and probe beams were passing through the acousto-optical modulators (AOMs) to make pulse time sequences and frequency scanning of the probe beams. For perfect spatial overlap, the 1st order of diffracted beams by AOM were coupled to the polarization maintenance optical fibers (PMF). The intensities of

probe beams were down to nW  $\sim$  pW by neutral density filter before PMF. Linear polarized coupling and probe beams were overlapped by PBS with an angle of  $2^\circ$ , and then, the beams were circularly polarized by quarter wave plate. After interaction with the trapped cold Rb atoms, probe beams were separated from coupling beams by PBS. BFs in front of the photomultiplier tubes were used to detect each probe beam separately (BF1(2) in Fig. 2: 795 nm (780 nm) band pass filter, BF1: Semrock, FF01-800, BF2: Semrock, LL01-780). We are using another auxiliary laser to pump another ground state  $|F=2\rangle$  which is turned off when probe laser is on. The intensity of probe laser is so weak that the population loss by probe laser during experiment for the signals is ignorable. For phase detection of probe beam, we adopt phase detection technique introduced in Ref. [27,28]. The 0th order of diffracted probe beams by AOM were overlapped with the 1st order diffracted probe beam before (and after) interaction with atomic medium. Beat notes generated by non-interacting with 1st order beams were used as reference beat notes. Comparing with those reference beat notes, we could observe the phase shift. We first observed transmission of CW probe beams P1, P2 in a double- $\Lambda$  and N1, 2 systems as a function of  $\delta_1$  ( $=\delta_2$ ).  $\delta_{1,2}$  was scanned by an AOM. Detuning of coupling beams were  $\Delta_{C1} = \Delta_{C2} = 3\gamma_3$ . Black line in Fig. 3(a) and (b) is transmission of P1 and P2, respectively. In this case, we fixed relative phase  $\phi_r = 0$  to make the condition of both probe fields being near the maximum transmission [24]. The transmission of P1, P2 was 0.77 at  $\delta_1 = 3.6$  MHz and 0.71 at  $\delta_1 = 1.9$  MHz. We could switch the double- $\Lambda$  to N-type system by turning off one of probe fields using BF1(2) before the atomic cloud instead of polarization changing of probe fields for experimental easiness. Red line in Fig. 3(a)(b) is P1(P2) transmission in N1 (N2) system. The transmission of P1 in N1 system was 0.54 at  $\delta_1 = 6.2$  MHz and P2 in N2 system was 0.44 at  $\delta_1 = 6.4$  MHz. Generated four-wave mixing signal of each N-type system [29,30] was 0.09 and 0.06 and considered as loss in this work. Dashed lines in Fig. 3 were the results of theoretical calculation with Eqs. (9) and (10). We, next, used  $1 \mu\text{s}$  probe pulse.

Transmissions of two probes are shown in Fig. 4(a) and (b). Phase shift was also measured and shown in Fig. 4(c) and (d). Fig. 4(a) and (c) for P1 and (b), (d) for P2. Black dots in Fig. 4 were experimental data in a DL and red dots were experimental results in N1, N2 system. The experimental parameters were the same to CW experiment except peak power of probe beams. The peak power of probe beams was  $\sim 4$  pW that corresponding to 16 photons per pulse. The transmission and phase

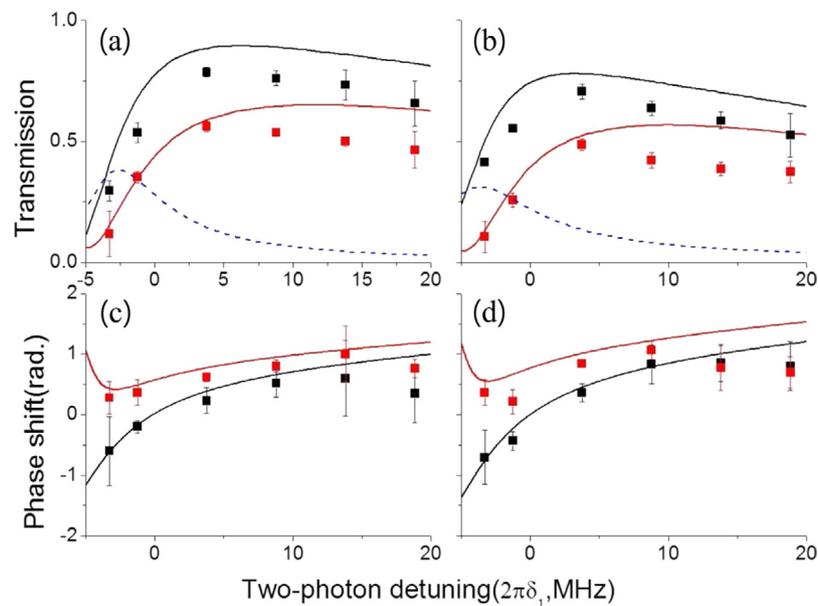


Fig. 4. Measured probe transmissions ((a) and (b)) and phase shifts ((c) and (d)). (a) and (c) for P1 and (b) and (d) for P2. Black lines are DL, and red lines are N1(2) system. Dots are experimental results and solid lines are theoretical calculation.

shift detection was 5000 times averaged signals. Maximum transmission of P1 was 0.78 (DL), 0.55 (N1) at  $\delta_{1,2} = 3.7$  MHz, and phase shift, at the same time, was 0.25 rad. and 0.62 rad. where P2 transmission was 0.7 (DL), 0.48(N2), and phase shift was 0.4 rad. and 0.83 rad. On the other hand, phase shift of P1 was  $-0.6$  (DL),  $0.26$ (N1) with transmission of  $0.3$  (DL),  $0.12$ (N1) at  $\delta_{1,2} = -3$  MHz where phase shift of P2 was  $-0.7$  (DL),  $0.4$ (N2) with transmission of  $0.41$  (DL),  $0.1$ (N2). Larger phase shift could be obtained with  $\delta_{1,2} < -3$  MHz, however, probe transmission in N-type system was too small. Phase shift difference  $\Delta\phi^{P1} = \phi_{DL}^{P1} - \phi_{N1}^{P1} = -0.37$  ( $-0.86$ ) rad,  $\Delta\phi^{P2} = \phi_{DL}^{P2} - \phi_{N1}^{P2} = -0.43$  ( $-1.1$ ) rad. with  $\delta_{1,2} = 3.7$  ( $-3$ ) MHz, consequently, C-XPM  $\phi_m$  was  $-0.9$  ( $-1.96$ ) rad. The transmission between ours and PRL 117, 203601 (2016) [19] is roughly same accidentally at some points. However, if optical depth is increased to have higher gate efficiency, the transmission at that point will be changed because of group velocity mismatching, and then PRL 117 might be difficult to have good transmission and the gate efficiency will be in trouble with different propagation velocity.

#### 4. Conclusion

We studied conditional cross-phase modulation via double- $\Lambda$  to N-type conversion by turning on/off one of probe fields. Turning off the one of two probe fields breaks multi-photon interference in an optical closed loop system of a double- $\Lambda$  system. This causes changing the mechanism of interaction between two probe fields. For actual logic gate application with photon, two probe pulses were group velocity matched with small difference ratio 1.2. To avoid strong suppression of probe transmission in N-type system, we established one-photon detuned system. We could increase C-XPM more than 1 rad. with small probe absorption, where probe pulse contained 16 photons. By the characteristic of double- $\Lambda$  system, we could expect the same results between less than 16 photons.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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